OR/MS Models for Supply Chain Disruptions: A Review

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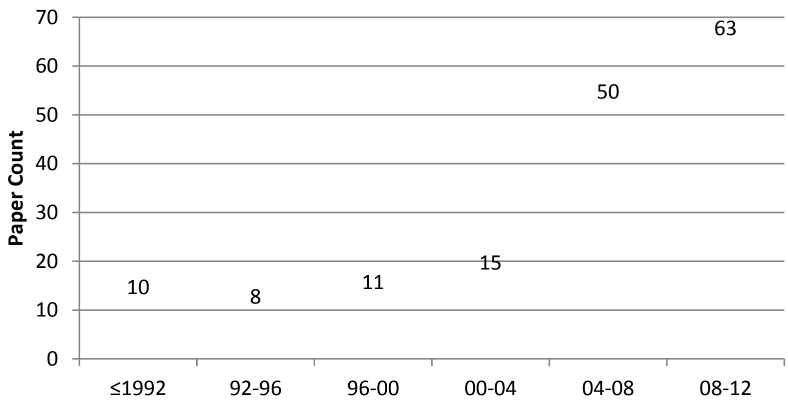
Abstract

We review the OR/MS literature on supply chain disruptions in order to take stock of the research to date and to provide an overview of the research questions that have been addressed. We first place disruptions in the context of other forms of supply uncertainty and discuss common modeling approaches. We then discuss nearly 150 scholarly works on the topic, organized into six categories: evaluating supply disruptions; strategic decisions; sourcing decisions; contracts and incentives; inventory; and facility location. We conclude with a discussion of future research directions.

1 Introduction

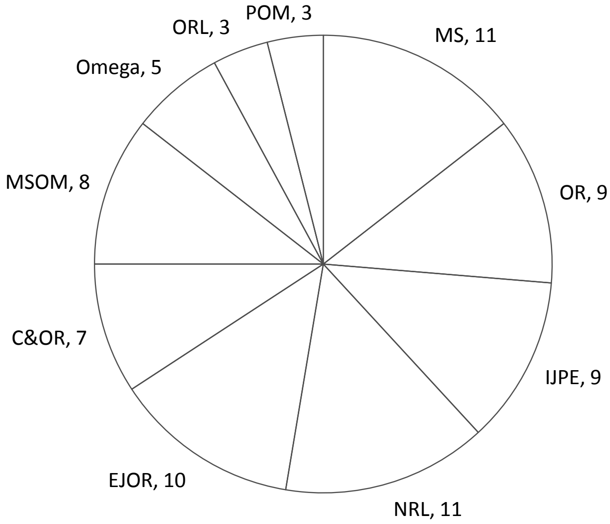
In the past decade, academics and practitioners have become increasingly interested in supply chain disruptions, which are caused by both natural disasters (floods, earthquakes, etc.) and intentional or unintentional human actions (industrial accidents, terrorist strikes, etc.). Academic journals and supply chain trade magazines have published numerous articles on the topic; universities have held workshops to discuss it; and *OR/MS Today* published a cover story about it (Clausen et al. 2001). The academic literature on supply chain disruptions has increased sharply during this time; see Figure 1 for a histogram of the dates of disruption-related works cited in this review. These papers have been published in the top journals in the field; see Figure 2.

Figure 1: Histogram of dates of disruption-related works (journal articles, working papers, books, etc.) cited in this review.



Supply chain disruptions are not new, of course; they have existed as long as supply chains have. So why the recent explosion of interest? We believe there are four main reasons. First, several high-profile events, including the terrorist attacks of September 11, 2001, the west-coast port lockout in 2002, and

Figure 2: Breakdown of journals in which disruption-related papers cited in this review were published. Working papers, books, and journals with fewer than three cited articles are excluded.



Hurricane Katrina in 2005, brought disruptions into the forefront of public attention. Second, some practitioners and researchers have concluded that the just-in-time (JIT) philosophy popular in recent decades increases supply chains' vulnerability to disruptions, since a tightly optimized, lean design, which performs well under normal situations, leaves little room for error when circumstances change drastically. Third, firms are much less vertically integrated than in the past, and their supply chains are increasingly global; suppliers are located throughout the world, some in regions that are politically or economically unstable. Finally, the topic has simply gained a critical momentum; like any maturing research area, scholars study the topic because their interest is sparked by emerging research and by increased attention to the topic in industry.

It is tempting to think of supply chain disruptions as rare events. However, while a given type of disruption (earthquake, fire, strike) may occur very infrequently, the large number of possible disruption causes, coupled with the vast scale of modern supply chains, makes the likelihood that some disruption will strike a given supply chain in a given year quite high. (Some supply chains face disruptions nearly every day; Wal-Mart even has an emergency operations center dedicated to preparing for and mitigating the effects of man-made and natural disasters.)

Even a small disruption can have a devastating impact as it cascades through a supply chain. For example, in 1998, strikes at two General Motors parts plants led to the shutdown of 100 other parts plants, then 26 assembly plants, leaving dealer lots vacant for months (Simison 1998). The economic impact of disruptions can be enormous. In a series of empirical studies, Hendricks and Singhal (2003, 2005*b*, 2005*a*) examine several hundred supply chain "glitches" reported in the *Wall Street Journal* and the Dow Jones News Service in the 1990s. They find that companies that experienced even minor disruptions faced significant declines in sales growth, stock returns, and shareholder wealth, and that these effects tended to linger for at least two years after the disruption.

The purpose of this review article is to take stock of the literature that has appeared to date and to provide an overview of the research questions that have been addressed. Our focus is on papers that use operations research/management science (OR/MS) methods to address supply chain disruptions. (We define the scope of our review more precisely in Section 1.2.)

In the subsections that remain in Section 1, we place disruptions in the context of other forms of supply uncertainty, delineate the scope of our review, and discuss the ways in which disruptions are typically modeled. Sections 2–7 discuss the literature, organized into six categories: strategic decisions; evaluating supply disruptions; sourcing decisions; contracts and incentives; inventory; and facility location. Finally, we summarize in Section 8 and suggest directions for future research.

1.1 Forms of Supply Uncertainty

Several forms of supply uncertainty have been discussed in the literature. *Disruptions* are random events that cause a supplier or other element of the supply chain to stop functioning, either completely or partially, for a (typically random) amount of time. In most models, disruptions affect a firm's supplier; during a disruption, the supplier cannot provide any goods. Under *yield uncertainty*, the quantity delivered by a supplier or produced by a manufacturing process is a random variable that depends on the order quantity. A closely related concept is *capacity uncertainty*, in which the supplier's delivery capacity or the firm's manufacturing capacity is a random variable that is typically independent of the order quantity. *Lead-time uncertainty* represents stochasticity in the order or processing lead time. *Input cost uncertainty* represents stochasticity in the procurement prices incurred by the firm.

The boundaries among these forms of supply uncertainty are often blurry. In fact, disruptions can often be viewed as a special case of yield uncertainty in which the yield is a Bernoulli random variable. However, yield uncertainty generally implies a continuous form of uncertainty, whereas disruptions are discrete, and therefore the two are often quite different in terms of both the approach toward modeling them and the managerial insights gained from them. The same is true for capacity uncertainty and lead-time uncertainty.

1.2 Scope of Review

We have attempted to provide as complete a review as possible on the topic of supply chain disruptions. Undoubtedly, we have inadvertently omitted some relevant papers, and for that we apologize both to our readers and to the authors of the overlooked papers. In order to limit this article to a manageable scope, we have intentionally omitted several important, related topics:

1. *Other Forms of Supply Uncertainty.* Although disruptions are sometimes viewed as a special case of yield, capacity, or lead-time uncertainty, because of the modeling and managerial differences between them (see Section 1.1), we have opted to omit the literature on these forms of supply uncertainty, except papers that deal with both disruptions and another form of supply uncertainty, and a few others. We refer the interested reader to the reviews on yield uncertainty by Grosfeld-Nir and Gerchak (2004) and Yano and Lee (1995); for a textbook treatment of lead-time uncertainty, see Zipkin (2000).

2. *Supply Chain Risk Management.* Risk management is a broad topic that encompasses not only disruptions but also other forms of supply chain risk, including financial and operational risks. In this paper, we consider only disruption risks, in the form of supplier unavailability, inaccessible facilities, production interruptions, lost capacity, and so on. See Tang (2006*a*) for a review of supply chain risk management more broadly.

3. *Network Reliability.* The facility location problems discussed in Section 7 are related to the literature on network reliability, which attempts to calculate the probability that a network remains connected after disruptions, or to optimize the design of a network with a connectivity objective in mind. Network reliability models sometimes consider the cost of constructing a network but, unlike the models in Section 7, usually ignore the increased routing cost that results from a disruption; that is, they focus more on connectivity than on cost since they are usually applied to power or telecommunications networks rather than transportation networks. We refer the reader to the review by Kerivin and Mahjoub (2005) and the textbooks by Colbourn (1987), Shier (1991), and Shooman (2002).

4. *Machine Breakdowns.* Machine breakdowns are a type of disruption and, like other disruptions, can have an impact throughout the supply chain. There is an extensive literature on machine breakdowns in the context of both scheduling and queuing problems. We omit these papers from our review since they are generally concerned with evaluating the performance measures of such systems, rather than optimizing their policy parameters, and since they usually consider frequent, and often planned, disruptions such as routine maintenance.

Other reviews and surveys of supply chain disruptions exist. Vakharia and Yenipazarli (2008) review the OR/MS and qualitative literature on supply chain disruptions. Natarajarathinam et al. (2009) review current practices and some quantitative research on managing disruptions. Schmitt and Tomlin (2012) review models for sourcing mitigation (see Section 4 below), while Atan and Snyder (2010) review models for inventory mitigation (see Section 6 below). Snyder et al. (2006) and Snyder and Daskin (2007) review models for facility location and network design with disruptions (see Section 7 below). Vanany et al. (2009) review and classify literature on supply chain risk management, focusing primarily on management journals. Tang (2006*a*) also reviews supply chain risk management more broadly, including some papers on disruptions in particular. The textbook by Yu and Qi (2004) discusses mathematical programming models for disruption management strategies in scheduling and supply chain contexts. The book by Murray and Grubesic (2006) discusses critical infrastructure reliability from a geographical modeling perspective. To our knowledge, ours is the most comprehensive review to date of OR/MS models for coping with supply chain disruptions.

1.3 Modeling Disruptions

Most of the papers discussed in this review model disruptions in an abstract way and are applicable to a wide range of disruption types and characteristics. The most common way that disruptions are modeled is to assume that the supply process has two states, one in which it functions normally and one in which it is disrupted. We will refer to the former state as the "up" state and the latter as the "down" state.[1] Typically the supply capacity is infinite during up periods and is zero during down periods, though some papers consider in-between cases in which disruptions do not completely eliminate the capacity, or in which the supplier has finite capacity even during up periods.

Typically, the system remains in the up state for a random amount of time, and then in the down state for a random amount of time. The most common assumption is that the duration of both the up period and the down period are exponentially distributed (in the continuous-time case) or geometrically distributed (for discrete-time models). Under this assumption, the supply process forms a two-state Markov chain whose parameters are generally referred to as the *disruption* and *recovery rates* (for continuous-time models) or the *disruption* and *recovery probabilities* (for discrete-time models). This simple supply model is flexible enough to accommodate a spectrum of *disruption profiles* ranging from infrequent-but-long to frequent-but-short. (Two disruption processes may have the same steady-state probability of being disrupted but may have very different disruption profiles.)

Some papers use a simpler supply model in which, for example, the disruption length is deterministic or the supply process is Bernoulli (a special case of the two-state Markov model in which the probability of disruption in the next period is the same whether the current period is disrupted or not). Other papers use more general distributions such as Erlang-k or even allow the distribution to remain unspecified. The vast majority of the papers discussed in this review use some variation of this supply model, although exceptions exist. For example, Snyder and Tomlin (2008) allow the disruption probabilities themselves to change in a Markovian manner over time, and Song and Zipkin (1996) assume that orders progress through the supply process randomly over time using a Markov model that is general enough to accommodate disruptions, random lead times, and so on.

The literature uses a wide range of assumptions about what part of the supply/replenishment process is disrupted. Can the firm place orders during a disruption? Can it receive orders? Can orders be shipped

[1]Several other terms are in common use; for example, wet/dry, on/off, normal/disrupted. We have changed the terminology used by many papers in order to consistently use the up/down terminology.

from a disrupted supplier if it has sufficient inventory? Does a disruption affect items that are already in transit? These questions are answered differently in each paper; no common convention has yet emerged. Some papers treat disruptions as though they disable the supplier: no orders can be placed, but orders that have been shipped to the firm but not yet received are unaffected. Others treat disruptions as though they occur in the transportation process: orders can be placed but not shipped, and items in transit are "paused" until the disruption ends. Still others assume that disruptions occur at the firm itself, effectively pausing all inbound and outbound replenishment activities.

2 Evaluating Disruptions

In this section, we discuss papers whose aim is primarily to evaluate disruptions and their effects, rather than to optimize the supply chain. These models generally aim to provide planners with tools to evaluate a given supply chain configuration (locations, capacities, etc.) under the risk of disruptions; some also consider one or more mitigation strategies.

Simulation is a natural tool for evaluating the impact of disruptions in complex supply chains. Deleris and Erhun (2005) introduce a simulation model of a multi-product, multi-echelon supply chain. Disruptions are modeled by scenarios, each with a different probability and severity. The model uses Monte Carlo simulation to determine the probability distribution of the loss of volume (compared to a no-disruption scenario) through the network. The motivation is to use the model as a strategic planning tool to evaluate different facility configurations, capacity levels, etc. Wilson (2007) simulates a five-echelon supply chain subject to transportation disruptions, whose impact is measured by stockout levels, inventory fluctuations, and behavior of goods in transit. The author simulates both a traditional supply chain and a vendor-managed inventory (VMI) system in which more demand information is shared upstream. She finds that disruptions have the greatest impact when they occur near the middle echelons of the supply chain, and that VMI dampens some, but not all, of the effects of disruptions.

Schmitt and Singh (2011) describe a simulation model for a three-echelon supply chain subject to disruptions, motivated by a study performed for a consumer packaged goods (CPG) company. They use their model to derive a number of qualitative insights. For example, the reaction speed of a backup resource is more important than its capacity; upstream disruptions have a longer impact on customers than downstream ones, and the impact on customers may last significantly longer than the disruption itself; and, demand spikes can have an even greater impact on the system than disruptions. Schmitt and Singh (2009) present a similar model that uses Monte Carlo simulation to estimate the risk profiles throughout the system and discrete-event simulation to model disruptions and material flow. The objective is to evaluate the impact of disruptions on service levels and the efficacy of various mitigation strategies. One insight that emerges from the analysis is that the service level is highly dependent on the inventory level at the onset of a disruption, and therefore the overall risk level changes constantly over time.

Li et al. (2006) model a supply chain as a directed acyclic supply network and consider disruptions on subgraphs of the network. The objective is to evaluate the impact of upstream disruptions on downstream stages in terms of time (when do they feel the disruption?) and cost (how severe is the impact?). The time question is evaluated by solving shortest-path problems, while the cost question is evaluated using formulas derived in the paper. The authors present an algorithm for answering both questions simultaneously using a single pass through the network, whereas a brute force approach may require many more iterations.

The impact of supply disruptions is not limited to the immediate downstream party. Rong, Shen and Snyder (2009*b*,*a*), Rong, Snyder and Shen (2009) postulate the existence of the "reverse bullwhip effect" (RBWE), which describes an increase in the order variance as one moves downstream from the disruption. (The RBWE is the opposite of the classical bullwhip effect (BWE), in which order variability increases in the opposite direction.) By using both a live "beer game" experiment and a simulation study, with supply uncertainty occurring at the manufacturer end of the supply chain, Rong, Shen and Snyder (2009*a*) show that the RBWE may result from either a player's reaction to supply disruptions (e.g., over- or under-ordering) or an over-emphasis on in-transit inventory in the player's order-quantity decision. (See also Ellis et al. (2010) for a behavioral study of purchasing decisions under the risk of disruptions.) Rong, Shen and Snyder (2009*b*) and Rong, Snyder and Shen (2009) analyze operational causes of the RBWE under supply uncertainty, namely, competition among retailers for scarce supply and pricing during disruptions with limited information about strategic customers (respectively). Rong, Snyder and Shen (2009) prove that the BWE occurs between retailers and customers and that the RBWE occurs between suppliers and retailers in the well known "rationing game" when there is limited or uncertain capacity at the supplier's end. Rong, Shen and Snyder (2009*b*) show that when customers react not only to the price itself but also to changes in price, a pricing mechanism implemented by the supplier that does not capture the underlying customer behavior may lead to the RBWE. These studies highlight the importance of treating the supply chain as an integrated system, since both supply and demand uncertainty can be magnified for the remaining parties.

The papers discussed so far, and in fact most papers on supply disruptions, assume that the firm knows the disruption process exactly. But it is often difficult to estimate disruption parameters accurately because of their rare occurrence and lack of historical data, and because suppliers are often unwilling to share disruption information with their customers. This is true when disruptions are modeled using the simple two-state Markov chain described in Section 1.3, but the problem is even more acute for disruption processes that have more complex forms (Song and Zipkin 1996, Li et al. 2004, Lewis et al. 2005). Several studies (e.g., Ross et al. 2008, Saghafian and Van Oyen 2012, Snyder and Tomlin 2008, all discussed below) demonstrate the value of having accurate information about supply disruptions. Therefore, it is necessary to investigate good estimation methods; otherwise the benefit of a model that assumes accurate information is undermined. Two ways to deal with this problem have been suggested in the literature. One is to design a forecast method; the other is to design a mechanism that induces the supplier to reveal its disruption characteristics. Tomlin (2009*b*) takes the former approach, using a Bayesian model to update the firm's forecast of the reliability of its supplier and deriving the optimal policy when the forecast is incorporated into sourcing and inventory decisions. The latter approach is taken by Yang et al. (2008), who assume that the supplier has private information about its reliability. The manufacturer offers the supplier a menu of contracts, consisting of a transfer payment, an order quantity, and a penalty cost, and the supplier chooses the contract that maximizes its profit. By applying mechanism theory, the authors show that the manufacturer can induce the supplier to report its reliability level truthfully.

3 Strategic Decisions

Disruption planning is often most effective when begun at the strategic level. Strategic decisions have a large impact on the functioning of the supply chain as a whole, as well as on individual locations, especially under the threat of disruptions. In this section, we review papers that discuss strategic decisions that can help protect supply chains against disruptions. The papers in Section 3.1 classify strategies for protecting against disruptions and offer guidance to firms seeking to choose among them. In Section 3.2 we discuss papers that examine what types of supply chain topologies are most resilient to disruptions. Section 3.3 discusses papers that consider the use or value of advanced information. (Another important strategic decision—facility location—has been studied extensively in the disruption literature and is discussed in its own section, Section 7.)

3.1 Strategies for Mitigating Disruptions

Tomlin (2006) discusses three categories of strategies for coping with supply chain disruptions: inventory control, sourcing, and acceptance. Inventory control strategies involve ordering and stocking decisions and can be considered proactive techniques. Sourcing strategies are reactive to an actual shortage; they can include product substitution (demand management) or backup supply (dual sourcing). Under the acceptance strategy, the firm does nothing to protect against disruptions and simply accepts the disruption risk, and the associated cost; typically, this strategy is favored when the cost of coping with disruptions outweighs the losses from accepting them. Tomlin examines the optimal choice (or mix) of strategies in a single-product system with two suppliers: one that is reliable and may have flexible capacity, and one that is unreliable but is less expensive. He demonstrates that the two main factors that determine the optimal strategy are the supplier's percentage uptime and the disruption profile (frequent–short vs. rare–long). He proves, for example, that if the unreliable supplier has infinite capacity and the reliable supplier is not flexible (i.e., cannot increase production levels quickly), then inventory (or acceptance) is the optimal mitigation strategy under frequent–short disruptions but sourcing is preferred for rare–long disruptions. If the reliable supplier is flexible, then a contingent rerouting strategy, in which supply volume is shifted to the reliable supplier only when the primary supplier is disrupted, may be optimal.

Tomlin (2009*a*) discusses optimal mitigation strategies when the product is perishable and therefore inventory is not a possible mitigation strategy. The strategies modeled are supplier diversification (routinely sourcing from multiple suppliers), contingent sourcing (switching suppliers if a disruption occurs), and demand switching (encouraging customers to purchase alternate products if a disruption occurs). Tomlin considers both risk-neutral and risk-averse decision makers and examines how the optimal strategy changes as the model parameters (costs, supplier reliability, and demand characteristics) change. He finds that as demand risk (as measured by coefficient of variation (CV)) increases, demand switching becomes preferable, but as supply risk (as measured by supplier reliability) increases, dual and/or contingent sourcing become preferable.

Schmitt (2011) considers a multi-echelon distribution system in which disruptions can occur on the links. The objective is to characterize the impact on service levels of various mitigation strategies, including inventory, backup capacity within the network, and external backup capacity. Demand is deterministic with partial backorders. Schmitt derives a general formula that describes the service level for any given combination of the mitigation strategies, first for a one-warehouse, multi-retailer (OWMR) system and then for a general distribution system. Numerical studies suggest, for example, that firms should use a mix of proactive and reactive mitigation strategies—inventory protects against the early periods of a disruption while backup supply protects against later ones.

Chopra and Sodhi (2004) and Kleindorfer and Saad (2005) discuss qualitative factors related to disruption-mitigation strategies. Chopra and Sodhi (2004) categorize types of risk and their drivers and assess the directional impact of various mitigation strategies. Kleindorfer and Saad (2005) classify the steps needed to address supply disruption risk and conclude that the two most important principles for disruption management are reducing the frequency and severity of the disruptions themselves and improving the supply chain's (and its players') ability to withstand disruptions without serious adverse consequences. Along similar lines, Tang and Tomlin (2008) suggest that reducing the impact of disruptions requires adding flexibility to the supply chain, such as flexible pricing or flexible sourcing. They suggest that investment in a small degree of flexibility can result in a large reduction in the magnitude of a disruption's impact.

3.2 Supply Chain Topologies

In this section we discuss papers that study characteristics of supply chain designs that make them more or less resilient to disruptions. These papers do not consider network design in the classical optimization sense (i.e., choose nodes and arcs to construct a network), but rather consider high-level questions comparing different network topologies.

Snyder and Shen (2006) investigate how demand uncertainty (DU) and supply uncertainty (SU) (in the form of disruptions) affect the optimal policies or topology of several multi-echelon supply chains. Using simulation, they demonstrate that the optimal strategy for coping with SU is often the exact opposite of the strategy for DU and argue that the two forms of uncertainty are, in a sense, mirror images of each other. For example, they show that in an OWMR system, a decentralized design (in which inventory is held at the retailers) is preferable, to reduce the impact of any one disruption, a phenomenon they call the "risk-diversification effect." In contrast, under DU, the classical risk-pooling effect (Eppen 1979) says that a centralized design is preferable. Snyder and Shen demonstrate that small increases in inventory can often provide substantial protection against disruptions.

Schmitt et al. (2011) analyze the risk-diversification effect in greater detail and prove that the expected costs of a centralized and decentralized design are equal under SU but that the cost variance is smaller in the decentralized system, making this the preferred design. (In contrast, under DU, the variances are equal but the centralized system has a smaller expected cost.) The authors also demonstrate numerically that, if SU and DU are both present, the risk-diversification benefits of decentralization typically trump the risk-pooling benefits of centralization, unless the service level is very low, the firm is very risk neutral, or the system is very reliable. (A phenomenon similar to the risk-diversification effect is noted by Pang and Whitt (2009) in the context of service systems. They show that the economies of scale to be gained from large-scale call centers are, in some cases, partially offset by the increased risk of congestion when disruptions, e.g., power outages, occur. This risk would be smaller in a system with multiple, smaller centers than a system with a single large one.)

Lim et al. (2011) use the "chaining" concept of Jordan and Graves (1995) to explore the value of flexibility in systems with disruptions. They model a multi-plant, multi-product system using a bipartite graph in which one set of nodes represents the plants, the other represents the products, and a link connects one node from each set if the plant can manufacture the product. Demand is random, and the system is subject to both link failures (in which a given plant can no longer produce a given product) and node failures (in which the plant cannot produce anything). The authors examine Jordan and Graves's claim (for their model without disruptions) that a single long chain is preferable to multiple short ones. They confirm this claim when the system is subject to node failures but find that shorter chains are preferred when link failures are the issue. The relevant performance measure is the system's "fragility," defined as the difference in expected lost sales before and after a disruption.

Hopp and Liu (2006) consider the placement of inventory and capacity in an assembly supply chain with disruptions and deterministic demand. The objective is to minimize the sum of the protection cost and the expected lost revenue from disruptions. The authors prove that it is optimal to have backup inventory or capacity at no more than one node on each path from a disruptable node to the customer, and that as the severity of the disruptions at a given node increases, the capacity or inventory buffers shift upstream toward that node.

3.3 Advanced Information

Since disruptions are typically rare but can have drastic effects, early warnings or other advanced information can play an important role in mitigating them. Snyder and Tomlin (2008) discuss how inventory systems should take advantage of advanced warnings of disruptions. They consider a system with an unreliable supplier (subject to complete disruptions) and a second, perfectly reliable supplier. The disruption characteristics, which constitute what Snyder and Tomlin call the "threat level," change stochastically over time, and the firm knows the current threat level. (One example is the U.S. Department of Homeland Security's "terror alert" level.) They show that the optimal inventory policy is a state-dependent base-stock policy, and they find the optimal base-stock levels in closed form for special cases and using dynamic programming in general. They suggest that a threat-advisory system can provide substantial cost savings by allowing the firm to increase inventory levels when the disruption risk is high and run "lean" otherwise.

Saghafian and Van Oyen (2012) also discuss the value of having advanced information about the status of an unreliable supplier when back-up supplier capacity is available. They quantify the impact of misperception of the unreliable supplier's status and the value of flexible versus dedicated suppliers. They indicate that advanced information is typically more valuable to a buyer than supply flexibility, particularly when the buyer has low profit margins.

Disruption risk is not the only form of uncertainty that can be reduced in advance. Wang and Tomlin (2009) consider advanced ordering by customers as a way to reduce demand risk when suppliers are unreliable. In their model, disruptions affect the lead time; when a disruption occurs (with fixed probability), the standard lead time is increased by a stochastic delay. The authors advocate allowing customers to place orders before the selling season and updating demand forecasts based on these orders. They indicate that while the timing of orders to the supplier is not affected by this pre-season ordering, the order quantity is, which provides cost savings to the buyer. They prove that the expected cost decreases as the ratio of pre-season orders to in-season orders increases.

4 Sourcing Decisions

Sourcing can be used as a disruption-mitigation strategy in two ways: routine sourcing (in which orders are placed from multiple suppliers simultaneously, before the supply uncertainty is resolved) and contingent rerouting (in which backup suppliers may be used if the primary suppliers are disrupted). We divide the literature on sourcing decisions into these two categories.

4.1 Routine Sourcing

Anupindi and Akella (1993) study the value of dual sourcing in the context of three models with supply uncertainty. Each model has a single- and a multi-period version. The first model assumes that each supplier delivers a given order immediately with a given probability, and otherwise it delivers the order in the next period (or never, in the single-period case). Thus, the supply uncertainty acts as a random 1-period disruption. The second and third models consider yield uncertainty. Demand is stochastic and has a continuous distribution. In all three models, and in both the single- and multi-period cases, they prove that the optimal ordering policy has three regions, based on the current on-hand inventory x: Order nothing (if x is large enough), order only from the less expensive supplier (if x is moderate), and order from both suppliers (if x is small). Swaminathan and Shanthikumar (1999) study Anupindi and Akella's first model and show that, when the demand distribution is discrete, rather than continuous, this ordering policy is no longer optimal. In particular, it is sometimes optimal to order only from the more expensive supplier if it is also more reliable. They also provide sufficient conditions under which it is optimal to order at least some units from the more expensive supplier.

Dada et al. (2007) consider a single-period newsvendor model with multiple unreliable suppliers. The supply process is quite general and allows the quantity delivered by a given supplier to be a function of both the order quantity and an exogenous random variable. Disruptions, yield uncertainty, and capacity uncertainty are all special cases. Demand is stochastic, with a continuous distribution. The objective is to choose which suppliers to order from and in what quantities in order to maximize the expected profit (sales and salvage revenues minus holding and stockout costs). The authors prove that, if a given supplier i is not used, then no more expensive suppliers than i will be used, suggesting that in a certain sense, cost trumps reliability. (This result generalizes that of Anupindi and Akella (1993).) Another conclusion is that if a given supplier is perfectly reliable, then no more expensive suppliers than that reliable supplier will be used. They also demonstrate that the optimal order quantity is larger and the optimal service level is smaller with unreliable suppliers than it is for the classical newsboy problem.

The model of Federgruen and Yang (2008) is similar to that of Dada et al. (2007). The suppliers are subject to yield uncertainty in the form of multiplicative yield with a general yield distribution. Disruptions (Bernoulli yield) are a special case. The cost structures of the two models are different: Dada et al. (2007) assume supplier-dependent variable costs, no fixed costs, and a stockout penalty, whereas Federgruen and Yang (2008) assume supplier-independent variable costs, supplier-dependent fixed costs, and a service level constraint. In both models, demand is stochastic. Federgruen and Yang's exact model is difficult to analyze, so the authors propose two approximations; one provides a provable upper bound on the total order quantity and cost, while the other is asymptotically optimal as the number of suppliers goes to infinity. The authors prove sufficient conditions for whether a given set of suppliers allows for a feasible solution; these conditions are based on the cardinality of the supplier set and on the CV of the suppliers' yield distributions.

Federgruen and Yang (2009) consider a modification of their 2008 model in which the fixed costs are zero and the per-unit costs are supplier dependent. The supply model is the same. The authors formulate two models, one using a service-level constraint and one using a stockout penalty. Both models use an approximation in which the ending inventory level is replaced by a normally distributed random variable with the same mean and variance. The key quantity in both models is the expected total yield from all suppliers, called the "expected effective supply." The authors derive closed-form expressions for the optimal order quantities for a *fixed* value of the expected effective supply and show that the total cost is a strictly convex function of the expected effective supply; therefore, the problem can be solved efficiently by searching over values for the expected effective supply. The authors prove that the optimal suppliers consist of the first k suppliers, for some k, after the suppliers are sorted in increasing order of their per-unit costs divided by their yield factors. This consecutivity result generalizes that of Anupindi and Akella (1993) and is also related to that of Dada et al. (2007).

Xu et al. (2011) consider a newsvendor model with multiple unreliable suppliers, like the previous three papers discussed. The key difference is that Xu et al. allow the supply uncertainty to be correlated among the suppliers, and they study the impact of this dependence. The authors consider two types of systems: a "multi-source" system in which the suppliers all produce the same part, and the newsvendor is trying to distribute his order among the suppliers to hedge against the supply uncertainty, and an "assembly" system in which each supplier produces a different part, and the newsvendor requires one of each part to produce the end product. The central result of the paper is that risk diversification is preferred in the multi-source system, but risk concentration is preferred in the assembly system. That is, in the multi-source system, the firm should choose suppliers that have independent disruptions in order to avoid having too many suppliers fall short simultaneously, whereas in the assembly system, the firm should choose suppliers that have positively correlated disruptions in order to avoid having too much inventory of one part when the supply of a complementary part is disrupted.

All four of these papers (Dada et al. 2007, Federgruen and Yang 2008, 2009, Xu et al. 2011) model a single retailer with N unreliable suppliers, but the nature of the supply uncertainty differs among them (although all can accommodate disruptions as a special case). The most general model is that of Dada et al. (2007), in which the quantity S delivered by a given supplier is given by $S = \min\{Q, K(Q,R)\}$, where Q is the order quantity for that supplier, R is a random variable, and $K(\cdot)$ is a function. This formulation is general enough to handle many models of supply. For example:

1. *Perfect Supply.* $K(Q,R) = \infty$
2. *Disruptions.* $K(Q,R) = R$; R equals 0 with probability α and ∞ (or some finite capacity level) with probability $1-\alpha$
3. *Multiplicative Yield Uncertainty.* $K(Q,R) = QR$; R has a continuous distribution with support $[0,1]$ (or a subset)

4. *Additive Yield Uncertainty.* $K(Q, R) = Q + R$; R has a continuous distribution with support $(-\infty, \infty)$ (or a subset)

5. *Capacity Uncertainty.* $K(Q, R) = R$; R has a continuous distribution with support $[0, \infty)$ (or a subset)

Federgruen and Yang (2008, 2009) use the multiplicative yield model ($K(Q, R) = QR$), while Xu et al. (2011) use the capacity uncertainty model ($K(Q, R) = R$).

Sawik (2011) considers a make-to-order system in which the customer demands are known in advance, and each requires customized parts that the firm procures from unreliable suppliers who are subject to both disruptions and yield uncertainty. The goal is to choose the order quantity for each supplier to minimize one of several objectives, including expected cost, worst-case cost, value-at-risk (VaR), and conditional value-at-risk (CVaR). Each model is formulated as a mixed-integer programming (MIP) problem and solved using an off-the-shelf solver.

Berger et al. (2004), Ruiz-Torres and Mahmoodi (2007) and Berger and Zeng (2006) consider a single buyer choosing from multiple identical suppliers subject to disruptions. Their model determines the optimal number of suppliers to use, assuming that the operating cost is a function of the number of suppliers and that a fixed penalty is incurred if all (or some) chosen suppliers fail simultaneously. Inventory costs are not modeled explicitly. The main conclusion of those papers is that the optimal number of suppliers is typically fairly small, except in very extreme cases where suppliers are very unreliable and the cost of failures is very high.

Yu et al. (2009) develop models to determine whether single or dual sourcing is optimal under non-stationary and price-sensitive demand. In the dual-sourcing case, the firm procures a fixed portion of the demand from the reliable supplier in every period and cannot place a second order after the unreliable supplier's state is observed. The authors develop closed-form expressions that characterize the conditions under which each strategy is optimal.

4.2 Contingent Rerouting

Tomlin and Wang (2005) consider a combination of dual sourcing and mix flexibility to cope with unreliable supply. In their model, the firm's suppliers are unreliable but also mix-flexible, i.e., can potentially produce multiple products. They consider four cases: single-source dedicated and flexible, and dual-source dedicated and flexible (in which order quantities are chosen after disruptions are realized). They show that the dual-source systems benefit from diversification while the single-source systems benefit from demand pooling and suggest that, surprisingly, a dedicated strategy may be preferred to a flexible one in some cases. Through a numerical study, the authors explore the tradeoffs among the network structures and examine the factors that affect the optimal decisions, including the firm's level of risk aversion, the number of products, and the product-demand correlations. They conclude that dual-source systems generally perform better when disruptions are a factor.

Chopra et al. (2007) argue for the importance of correctly identifying, and planning for, the forms of supply uncertainty a firm faces. In their model, the firm has two suppliers, one reliable and one subject to both random yield and disruptions. Their model assumes a single period and deterministic demand. They derive closed-form expressions for the optimal order quantity from the unreliable supplier and the optimal reservation quantity from the reliable one. They demonstrate the error that occurs when the firm "bundles" the yield uncertainty and disruptions by treating the entire supply process as though it arises from yield uncertainty. They conclude that the retailer should determine its strategy depending on the source of the supply risk. The retailer should order more from the unreliable supplier if most of the supply risk comes from yield uncertainty, but it should rely more on the reliable supplier if most of the risk comes from disruptions. Schmitt and Snyder (2012) extend this model to a multiple-period setting and, like Chopra et al. (2007), determine the optimal order and reservation quantities. They show that approximating this system using a single-period model causes over-estimation of the cost, under-utilization of the unreliable supplier, and distortion of the order quantities.

In the standard contingent-rerouting strategy, if the firm wishes to order but the primary supplier is disrupted, it immediately orders from the backup supplier. In contrast, Qi (2009) allows the firm to wait a while in case the cheaper, primary supplier recovers quickly. The system is nearly identical to the economic order quantity (EOQ) model with disruptions introduced by Parlar and Berkin (1991) (see Section 6.1) except for the dual-sourcing aspect. The firm will wait for a certain duration, called the "cap," before ordering from the backup supplier, and the cap is a decision variable. (If the cap equals zero, we have the standard contingent-rerouting setup.) The primary conclusion of the analysis is that it is always optimal for the firm to either order from the backup supplier immediately after the safety stock runs out or wait as long as necessary until the primary supplier recovers.

Whereas the papers cited above consider a single location facing stochastic demand, Xu and Nozick (2009) consider a supplier-selection problem for multiple plants that face deterministic demand and order components from multiple unreliable suppliers. The firm may engage a supplier as a "base" supplier and pay a per-unit cost for each unit ordered or as an "option" supplier that will be used if the base suppliers are disrupted; option suppliers are paid a fixed reservation cost and a per-unit cost for each item actually ordered. Components are shipped from suppliers to plants via a capacitated multi-modal network, possibly using transshipment nodes. The problem is to decide which suppliers to use for each component, what type of relationship to establish with each, how large the option contracts should be, and how to route components through the inbound network, to minimize the expected total cost over the multi-period planning horizon. The authors formulate the problem as a two-stage stochastic program and propose an algorithm based on Lagrangian relaxation and the L-shaped method.

Motivated by the well known disruption case study in which a fire in 2000 at a Philips semiconductor plant caused supply disruptions for Ericsson, who reacted sluggishly, and Nokia, who responded aggressively and eventually stole significant market share from Ericsson, Hopp et al. (2009) introduce a model in which two competing firms (e.g., Nokia and Ericsson) share an unreliable primary supplier (Philips) and a backup supplier that has limited capacity. A third, exogenous firm produces an inferior product but may steal market share from the first two if they cannot meet their demand. The two firms' key decisions are how much to invest in backup capacity (both before and after a disruption) and how much to invest in disruption detection. The authors derive a range of managerial insights both analytically and numerically; for example, firms with large market share should focus on protecting it while smaller firms should be poised to steal market share from larger, disrupted firms.

5 Contracts and Incentives

When supply disruption risk exists, financial means may be employed to reduce that risk or increase the number of suppliers willing to compete for demand. Conversely, financial problems themselves often create supply risk. There is a growing body of literature that models how contractual relationships and risk sharing can decrease operational supply risk, and how supply risk affects contract and pricing negotiations. In this section, we discuss papers that focus on these contracting and incentive issues.

A supplier bankruptcy can serve as a disruption if the supplier can no longer provide raw materials. If alternate supply options are not available, a buyer may choose to invest in the supplier in order to ensure its economic stability. These investments may include providing monetary subsidies or assuming asset ownership. Babich (2010) models the former case, showing how a supplier's financial state is related to its ability to fulfill orders and how a buyer can improve its supply risk by reducing the supplier's financial risk. The model assumes that a supplier knows its financial risk state, and that this state relates to the capacity it has available to meet the buyer's demand. The supplier shares this information with the buyer, at which time the buyer may choose to subsidize the supplier. Subsidy funds may be used for liability reduction or asset investment, either of which improve the probability that the supplier can meet demand. For the case where subsidies are invested in assets, Babich shows that optimal order quantity decisions should be made independently of subsidizing decisions, and that the buyer's optimal subsidizing policy is of a "subsidize-up-to" structure; that is, a buyer should bring its supplier's asset level up to an optimal threshold value.

Contract design under supply risk relies heavily on the players' knowledge of, or beliefs about, the supplier's reliability level. Gurnani and Shi (2006) model a first-time interaction between a buyer and an unreliable supplier; reliability is measured by the probability that the supplier will deliver the agreed-upon quantity by the agreed-upon date. Since this is the first interaction between the two, the reliability level is unknown, and both parties must estimate it. There are two contracts available, one that involves a down-payment (incurred even if the supplier is unable to deliver) and one in which the buyer pays only upon delivery. The authors characterize the cases in which the players prefer each contract type and explore the ability of each to maximize channel profits.

In some cases, the supplier may know its own reliability level, and the buyer may be willing to pay for that information. Yang et al. (2008) assume the supplier is obligated to pay a penalty if a disruption occurs and it cannot meet the retailer's orders. When information is asymmetric and only the supplier knows its risk status, the buyer may offer multiple contracts to the supplier and allow it to choose among them based on its risk level. Alternately, the buyer may pay to make the risk level information symmetric prior to offering the supply contract. The authors demonstrate that if backup supply is expensive, the buyer is more willing to pay for this risk-level information. Gao and Li (2010) also advocate using contracts to encourage the supplier to share information about its risk level. They prove that when fixed ordering costs are present, a state-dependent (s, S) policy is optimal if information on the state of the supplier is available. They demonstrate that fixing costs and fees in contract negotiations encourages a supplier to share its state information and can coordinate the supply chain.

Auctions are another mechanism for inducing suppliers to reveal their reliability levels, as well as for choosing from among multiple unreliable suppliers. Chaturvedi and Martínez-de Albéniz (2011) introduce an auction mechanism in which a single buyer wishes to decide which suppliers to use and how much to procure from each. The authors consider the case in which suppliers' cost and reliability are known to the buyer, then the case in which only the reliability is known, and finally the case in which neither is known. They propose sealed-bid auction mechanisms that induce truth-telling by the bidders; they involve two sets of payments, one before and one after delivery. The authors show that the optimal strategy usually involves diversification among the suppliers, a finding echoed by Dada et al. (2007) and related papers (see Section 4.1).

If a supplier is known to be at risk, the buyer may insist on a contract that requires the supplier to reimburse it for some of its costs if a disruption occurs and causes a material shortage. If such a disruption occurs for commodity products, spot markets may also be used to rectify the supply shortage and meet customer demand. However, spot markets, too, entail risk, in the form of fluctuating prices. Haksoz and Kadam (2008) develop simulation models to analyze the tradeoffs among the risks from supplier breach, spot market pricing, and demand fluctuations, emphasizing the importance of the firm decreasing its profit volatility. Their results suggest that if demand variability is high, it is better to under-order from the contractual supplier, since over-ordering can cause high profit losses. They also suggest that spot markets are a valuable tool to mitigate material shortage risk from both demand volatility and supplier breach.

Shou et al. (2011) examine coordination and competition in two simple supply chains, each consisting of a single retailer and a single supplier subject to Bernoulli disruptions. The retailers engage in Cournot (quantity) competition in a single selling season. The authors consider two types of contracts between retailer and supplier: a revenue-sharing contract and a contract in which the retailer pays a wholesale price for each unit delivered and the supplier pays a penalty for each unit ordered but not delivered.

The authors prove that the first contract coordinates the supply chain while the second does not. They consider three cases, in which both, one, or neither supply chain is coordinated, and they characterize the equilibrium order quantities for each case. They prove that, as the level of coordination increases, customers always benefit since the expected supply increases and therefore the expected price decreases. On the other hand, the supply chains themselves may not benefit (especially if the disruption probabilities are small), but coordination is nevertheless a dominant strategy for both supply chains.

A firm may wish to increase its supply base to reduce the impact of a supplier disruption, and it may use financial incentives to enable that option. Tang (2006*b*) discusses this approach, postulating that a buyer who feels it has too few suppliers may offer incentives for more suppliers to enter the market. These incentives could include committing to minimum order quantities or to sharing technical or market information in order to reduce potential suppliers' risks or barriers to entry. This strategy has the benefits both of diversifying supply disruption risk and of increasing the competition among suppliers and potentially lowering wholesale prices. Tang suggests other mitigation strategies to increase robustness in a supply chain, including using promotions or dynamic pricing to control and shift demand away from products that experience disruptions.

Suppliers themselves may also use financial means to compete for their share of a buyer's order despite their own unreliability. Swinney and Netessine (2009) discuss a single-period newsboy model with one unreliable supplier and a more expensive, perfectly reliable supplier that requires orders to be placed in advance. They solve for the reliable supplier's optimal pricing policy when it knows the buyer's behavior and competitor's risk level, and they discuss conditions under which a Nash equilibrium exists when both suppliers can set their prices competitively. Babich et al. (2007) develop a single-period model with pricing competition among multiple unreliable suppliers and show that this competition can create a diversified supply base for the buyer. They show that if suppliers' disruptions are correlated, then each supplier's competitive advantage is decreased and they must compete based on wholesale pricing. As positive disruption correlation increases, the buyer increasingly chooses to order from only one supplier (the cheapest), despite the reduced diversification benefits. However, if disruptions are negatively correlated among suppliers, the buyer will order from multiple suppliers and wholesale prices will increase as the suppliers are better able to compete with each other on reliability. As the total number of available suppliers increases, the buyer can realize benefits from both competition and diversification. Serel (2008) examines a similar model with competitive suppliers, extending the analysis to consider multiple periods and supplier default risk, which is reduced by the buyer's business. They show that while short-term contracts are normally preferred by the buyer, adding supplier default risk to the model typically changes that choice and drives long-term investment by the buyer in the supplier's business. They also consider dynamic contracts, in which the buyer shares some of the production cost risk with the supplier, and prove that long-term dynamic contracts can coordinate the supply chain. Li et al. (2010) consider a two-supplier model that is similar to that of Babich et al. (2007), with the aim of finding equilibrium pricing and ordering decisions, as well as a coordination mechanism for a system in which the suppliers are cooperative with each other but competitive with the retailer.

Babich (2006) extends the case of positively correlated supplier disruptions to consider differing lead times for two suppliers. He examines price deferment on the part of the faster supplier, in which that supplier can wait to set its price until the slower supplier's disruption status is known. He shows that, somewhat surprisingly, this encourages the buyer to order more from the slower supplier as long as prices are moderate, because the faster supplier is treated as a backup source. Wagner et al. (2009) encourage the modeling of disruption dependence among suppliers, providing empirical evidence from the automotive industry demonstrating that supplier default risks are typically positively correlated.

Xiao and Yu (2006) study an evolutionary game (that is, a game in which players may not act fully rationally or may lack full knowledge of the rules of the game) consisting of a duopoly, with each firm operating one manufacturer and a large number of retailers. In one model, the manufacturer is subject to supply disruptions, while in another, the retailers are subject to demand disruptions. In each market, retailers from the two firms compete by choosing whether to pursue a profit- or revenue-maximizing strategy, and how much to order from the manufacturer. The aim is to determine the evolutionarily stable strategy (ESS) for each player, as well as the effect of disruptions on the ESS.

6 Inventory and Disruptions

The main concern in inventory management problems is to find the optimal replenishment policy, which indicates when, from whom, and how much to order. In most problems, the purpose is to minimize the expected cost, possibly subject to a constraint on the service level. For relatively simple problems, one can obtain closed-form solutions. Such solutions are attractive since they can be computed easily (e.g., in a spreadsheet), they often provide insights into the behavior of the system, and they are more easily embedded into more complex models. However, for many problems, especially problems with disruptions, closed-form solutions are not available; these problems must be solved algorithmically. Moreover, the form of the optimal inventory policy is unknown for many settings with disruptions. Therefore, many of the papers discussed below choose a reasonable inventory policy (often based on a classical inventory policy) and then seek the best parameters for that policy, without proving the policy is optimal.

Like the literature on inventory models without disruptions, models for inventory management under the threat of disruptions can be classified along a number of dimensions: periodic vs. continuous review, cost structures (e.g., fixed vs. variable costs), decision variables (e.g., order quantity, order-up-to level, reorder point), backorders vs. lost sales, number of suppliers, and so on. To this list we might add disruption distributions, since the papers cited below use a variety of approaches to model the duration of up and down periods. There are very few disruption-related papers in which decisions must be made for multiple echelons of the supply chain, and to our knowledge only two of these papers are interested primarily in inventory-theoretic questions. Those papers (Atan and Snyder 2012, DeCroix 2012) are discussed in Section 6.2 below; the others are cited elsewhere in this review. There is also a small body of literature on disruptions in multiple-supplier inventory systems, and a few of those papers are discussed below; others are interested in broader strategic questions rather than solely on inventory-theoretic questions and are discussed elsewhere in this review (e.g., in Section 4).

Treatment of supply uncertainty in production and inventory systems can be traced back to Meyer et al. (1979), who analyze a single-stage production-storage system facing a constant demand rate. When operable, the production facility is assumed to produce at a rate greater than the demand rate until the storage capacity is full. This serves as a precaution against the random failures of the supply source. For a particular distribution, exact expessions for the average inventory level and the reliability (defined as the fraction of time during which demands are met) of the system are obtained and the performance measures are shown to depend much more on the means of the on- and off-time distributions than on their types. This work is extended by Posner and Berg (1989) to the case in which demand follows a compound Poisson distribution.

In another early work, Chao (1987) models an inventory system subject to random disruptions as a continuous-time Markov decision process with a finite state space. The author presents a general characterization of the optimal inventory policy and interprets it in economic terms. For the same problem, Chao et al. (1989) use stochastic dynamic programming to find optimal inventory policies for electric utility companies which may face market disruptions.

Following these initial papers, many authors have studied inventory systems subject to disruptions. In what follows, we discuss the literature on continuous-review problems first, and then turn our attention to periodic-review problems. Most of the literature cited below considers a single supplier; the key questions in these models are when to order and how much. Typically, optimal order quantities are larger for problems with disruptions than for those without (see, e.g., Groenevelt et al. 1992*b*, Snyder 2012) since the extra inventory is used to provide a buffer against the extra uncertainty.

6.1 Continuous-Review Models

Parlar and Berkin (1991) are the first to introduce disruptions into the most basic continuous-review model, namely the EOQ model. This problem has come to be known as the EOQ with disruptions (EOQD). Demand is deterministic and continuous, demands that occur when the firm has no inventory are lost, and on and off periods have random lengths. The authors use the renewal reward theorem to derive an expression for the expected cost per unit time, which is the objective to be minimized. They prove that their cost function is convex for exponentially distributed on and off periods. However, their model was shown to be incorrect by Berk and Arreola-Risa (1994), who point out two implicit (and incorrect) assumptions in the original model: that stockouts occur during every off period (which is not true if the disruption ends before the inventory level hits zero) and that the shortage cost is incurred per unit time (which is incorrect for lost sales). The second assumption is easy to correct, but the first requires a redefinition of a "cycle" in the context of the renewal reward theorem. Berk and Arreola-Risa's corrected model, which assumes exponentially distributed on and off times, has a quasiconvex cost function but cannot be solved in closed form.

Following these two papers, many authors study the EOQD model. Bar-Lev et al. (1993) incorporate stochastic demand into the EOQD model of Parlar and Berkin (1991). In particular, the inventory process (which is driven by the demand) is assumed to be a Brownian motion with negative drift. The authors derive the expected cost as a function of the decision variables (order quantity and capacity) using renewal theory and minimize it numerically. Snyder (2012) introduces a simple but effective approximation to the optimal order quantity for the EOQD (as formulated by Berk and Arreola-Risa (1994)). The approximation replaces an exponential term in the model with a constant; as a result, the (near-)optimal order quantity can be found in closed form. Qi et al. (2009) extend the EOQD model to include disruptions both at the supplier and at the firm itself. The authors construct an effective approximation that uses a similar idea as that of Snyder (2012). Their numerical study suggests that disruptions at the firm have a larger impact on its cost and fill rate than disruptions at the supplier. Weiss and Rosenthal (1992) also study an EOQ inventory system in which a disruption, in either the supply or demand process, is possible at a single known time in the future; the duration of the disruption is random. The authors determine the optimal inventory policy for both cases (supply disruption and demand disruption) and present an iterative algorithm for finding the optimal order quantity. Ross et al. (2008) study the EOQD problem with on and off times that have phase-type distributions; the parameters of these distributions, and the mean of the Poisson demand process, are time dependent, following a cyclic pattern. The authors conclude that non-stationary policies not only provide some cost benefit but are also robust to errors in estimating the system parameters.

Groenevelt et al. (1992*a*) and Groenevelt et al. (1992*b*) consider an EOQD-type model applied to manufacturing processes. In this context, a disruption represents a machine breakdown and subsequent corrective maintenance. The authors investigate the effects of these disruptions on lot-sizing decisions. They show that the optimal lot sizes for a process subject to disruptions are always larger than those for the deterministic EOQ case.

In the classical EOQ model, it is well known that a zero-inventory ordering (ZIO) policy, in which an order is placed only when the inventory level reaches zero, is optimal. Such a policy is not necessarily optimal for the EOQD—it may be preferable to order before the inventory level reaches zero in case the supplier is disrupted at that point. (However, Bielecki and Kumar (1988) demonstrate that a policy similar to a ZIO policy is sometimes optimal for an unreliable manufacturing system, contradicting the common belief that inventories are always valuable as buffers in uncertain environments). Parlar and Berkin (1991) and Berk and Arreola-Risa (1994) assume a ZIO policy even though it is not necessarily optimal, but Parlar and Perry (1995) extend the EOQD model to allow the reorder point to be a decision variable.

Parlar and Perry (1996) also relax the ZIO assumption but assume that the state of the supplier can be ascertained at no cost; therefore, a straightforward (Q, r) policy is appropriate. The resulting model is solved numerically. They also consider models with two and with $N > 2$ suppliers. The authors propose a near-optimal ordering policy for this problem and solve it numerically. They prove that, as the number of suppliers goes to infinity, the problem reduces to the classical EOQ. Gürler and Parlar (1997) also consider a system with two suppliers and extend the work of Parlar and Perry (1996) by considering Erlang-k on times and generally distributed off times. Using the renewal reward theorem, they determine an expression for the average cost, which they solve numerically. Heimann and Waage (2007) extend the approximation by Snyder (2012) by relaxing the ZIO assumption so that it applies to the single-supplier model of Parlar and Perry (1996). They develop approximate closed-form expressions for the optimal order quantity and reorder point.

The renewal reward theorem is used in many disruption papers (e.g., Parlar and Berkin 1991, Berk and Arreola-Risa 1994, Parlar and Perry 1995, 1996) because it allows the expected cost per unit time to be calculated knowing only the first moment of the cost per cycle and the cycle time. Parlar (2000) argues that we may be interested in performance measures other than expected cost, in which case higher moments of these two random variables are required. To that end, the author derives the exact distribution and the moments of the cycle time and cycle cost random variables and uses these to calculate the probability that the cost per unit time will exceed a given threshold value.

Kalpakam and Sapna (1997) consider a system that is identical to the EOQD system except in two ways: (1) there is a per-unit procurement cost, which is often omitted in EOQD models, and (2) demands are discrete and follow a renewal process. The firm is assumed to follow an (s, S) policy. The authors analyze the steady-state behavior of the system and use this to formulate the expected cost function. Moinzadeh and Aggarwal (1997) also propose an (s, S) policy for an unreliable bottleneck production/inventory system with constant finite production and demand rates. The authors provide expressions to evaluate the operating characteristics of the system and analyze the properties of the expected total cost rate as the system parameters change. They develop a heuristic for finding the optimal inventory parameters. A similar problem with compound Poisson demand and general demand-size distribution is analyzed by Liu and Cao (1999). The authors derive an expression for the steady-state distribution of the inventory process. Arreola-Risa and DeCroix (1998) also consider a system managed with an (s, S) inventory policy. They consider partial backordering and obtain an exact closed-form expression for the cost function and propose an algorithm for computing the optimal policy parameters. In addition, they show how these optimal values change as the severity of the supply disruptions and the behavior of the unmet demands change.

Gupta (1996) considers stochastic demands but, unlike Bar-Lev et al. (1993), relaxes the ZIO assumption. The system is managed using a continuous-review (Q, r) policy. If the supplier is disrupted when the inventory position hits r, the corresponding order cannot be filled for the remainder of the supplier's disrupted period. Numerical results suggest that ignoring disruptions when choosing policy parameters may cause high operating costs, especially in an environment in which disruptions are long and the stockout cost is high. Mohebbi (2003) extends Gupta's (1996) model to consider both compound Poisson demands and stochastic (Erlang) lead times. In a subsequent paper, Mohebbi (2004) assumes that the supplier's on and off periods constitute an alternating renewal process. On and off periods are modeled as random variables following general and hyperexponential distributions, respectively. By deriving the stationary distribution of the on-hand inventory level, the author obtains an exact expression for the long-run average cost rate function. Parlar (1997), Mohebbi and Hao (2006) and Mohebbi and Hao (2008) consider models similar to that of Gupta (1996) but assumes that lead times are random.

Most inventory models with disruptions are primarily proactive: They determine optimal inventory policies to protect against future disruptions, assuming that few, if any, recourse actions are possible after a disruption. In contrast, the focus of the model by Xia et al. (2004) is to react to a disruption by developing a post-disruption production/inventory plan that is as close as possible to the pre-disruption plan. They consider a two-stage system in which the first stage functions as an economic production quantity (EPQ) system, manufacturing the product and storing inventory, while the second stage functions as an inventory location only. Disruptions manifest themselves by causing (possibly abrupt) changes in one of a number of parameters, including costs and production and demand rates. In response, the firm can adjust its production schedule. The problem is formulated as a quadratic programming problem.

6.2 Periodic-Review Models

Parlar et al. (1995) consider a finite-horizon, periodic-review system with random demand, zero lead time, and an unreliable supplier whose on and off periods are geometrically distributed. The probability that the current period's order is filled depends on the disruption state of the supplier in the previous period. A setup cost is assessed whenever an order is placed and another setup cost is incurred when the order is filled. The authors prove that the optimal inventory policy for this problem is of an (s, S) type. They show that the reorder level s depends on the state of the supplier in the last period but the order-up-to level S does not. Özekici and Parlar (1999) generalize the model of Parlar et al. (1995) by considering cost parameters that change in response to changes in an exogenous environment.

Song and Zipkin (1996) analyze a periodic-review inventory system with an exogenous Markovian supply process in which the state of the system may affect the lead time. The supply process is quite general and may be stochastic because of disruptions, stochastic lead times, etc. The authors prove that a base-stock policy is optimal if there is no fixed cost and an (s, S) policy is optimal otherwise; both policies are state dependent, with the optimal parameters depending on the state of the supply process. Because of the generality of the supply model, this result applies to a wide range of inventory problems with disruptions.

Güllü et al. (1997) consider a nonstationary supply uncertainty model, with a deterministic but dynamic demand sequence. Under a Bernoulli supply process and linear order costs, the authors introduce a dynamic programming formulation and use this to show that a base-stock policy is optimal. Moreover, they show that the solution has a newsboy-type form in which the optimal order-up-to level equals the sum of the next k period's worth of demands. Güllü et al. (1999) extend this analysis to a disruption process that allows for partial disruptions.

Argon et al. (2001), Li et al. (2004) and Lewis et al. (2005) also study periodic-review systems subject to disruptions. Argon et al. (2001) find the base-stock level maximizing the expected profit. Considering different assumptions Li et al. (2004) and Lewis et al. (2005) prove the optimality of state-dependent base-stock policies.

Yan and Liu (2009) present a periodic-review, finite-horizon model in which the firm has two suppliers and must decide the order quantity from each, as well as the sales price to the customer. One supplier is expensive, fast, and reliable (orders arrive instantaneously) and the other cheap, slow, and unreliable (orders arrive at the beginning of the next period, or not at all). The authors prove that the optimal policy is of the form $(s, S, p, \sigma, \Sigma)$, where p is the sales price, s and S are the reorder point and order-up-to level for the reliable supplier, and σ and Σ are analogous quantities for the unreliable supplier.

Schmitt et al. (2010) analyze the impact of supply disruptions combined with either demand uncertainty or supply yield uncertainty. Assuming a base-stock policy, the authors develop an approximation that treats all supply states except one as though they had deterministic demand (or yield) and, in doing so, replaces the loss functions in the expected cost with linear functions. The resulting cost function can be solved in closed form. The authors report that their approximation had an average error of less than 0.2% on their test instances.

Atan and Snyder (2012) generalize the model by Schmitt et al. (2011) (discussed in Section 3.2) by allowing inventory to be held at both echelons. Both studies develop approximations to solve for the base-stock levels which minimize the total system cost.

Under demand uncertainty, it is well known (Rosling 1989) that an assembly system (in which each node has at most one predecessor) can be reduced to an equivalent series system, and therefore that a base-stock policy is optimal and that optimal base-stock levels can be solved efficiently using the algorithm by Clark and Scarf (1960). Unfortunately, the equivalence between assembly and series systems does not hold when the nodes are subject to disruptions. This is proven by DeCroix (2012), who shows that the situation is in fact even more dire since a base-stock policy is not optimal for this system. The author does show, however, that an assembly system can be reduced to a simpler, equivalent assembly system in which parts of the system (or sometimes the entire system) are replaced with series systems.

7 Facility Location

The papers discussed in the preceding sections generally address the question *how much?*—how much inventory to store, how much to order from backup suppliers, how much to subsidize a supplier, etc. In contrast, the papers in this section address the question *where?*—where inventory should be stored and distributed from, and how it should be routed through the supply chain network.

When disruptions strike, the answers to these questions often change in real time. For example, Chiquita and other shippers were forced to reroute ships to other ports during the closure of the Port of New Orleans in the aftermath of Hurricane Katrina in 2005 (Journal of Commerce 2005). Similarly, during a 2009 strike at a distribution center in Edmonton, Safeway transferred some distribution activities to facilities in Calgary and Winnipeg, some 300 km and 1300 km away, respectively (Kleiss 2009).

Of course, only the recourse (e.g., rerouting) decisions can be adjusted after a disruption; usually, facility location decisions must be made far in advance and can be changed only infrequently. The two central questions, then, are whether the optimal set of facility locations should be different given that there is a risk of disruptions during the facilities' lifespan, and whether a more resilient set of facilities needs to be much more expensive. The short answers to these questions are "yes" and "no," respectively; the papers discussed below provide more details.

We divide the papers below into three sections. The first group of papers address the basic question of where to locate facilities to serve customers efficiently when disruptions occur (and, in some papers, when they do not). The second group is concerned with interdiction (the intentional disruption of a network) and the ways that facilities can be located and/or fortified in order to protect against interdictions or random disruptions. In the third section, we discuss papers that incorporate inventory management into facility location decisions.

7.1 Basic Models

The literature on deterministic facility location problems is extensive; for textbook treatments, see, e.g., Daskin (1995), Drezner (1995), and Drezner and Hamacher (2002). The first studies of stochastic location models appeared in the 1970s and 1980s, focusing primarily on random demands and costs, rather than on disruptions; for a review, see Snyder (2006). In this section, we discuss papers that assume that one or more facilities (or, less commonly, transportation arcs) may be disrupted. Some of these are also discussed, in more mathematical detail, in the tutorial by Snyder et al. (2006) and the book chapter by Snyder and Daskin (2007).

To the best of our knowledge, the first paper to consider disruptions in a facility location model is that of Drezner (1987), who introduces two models. The first, a reliability version of the classical p-median problem, assumes that nodes fail with a given probability. In the second model, called the "(p, q)-center problem," p facilities must be located to minimize the maximum cost that may occur when at most q facilities fail. Drezner proposes neighborhood-search-type heuristics for both problems. Lee (2001) discusses a continuous (planar) version of the unreliable p-median model and suggests heuristics based on space-filling curves.

Snyder and Daskin (2005) introduce the reliability fixed-charge location problem (RFLP) and the reliability p-median problem (RPMP). Both models use a bi-objective formulation in which one objective corresponds to the "nominal" cost (ignoring disruptions) and the other corresponds to the expected "failure" cost (i.e., the expected transportation cost, accounting for disruptions). Like the models of Drezner (1987), their formulations assume that customers are served by the nearest non-disrupted facility. By assuming that all facilities, except for a set of facilities that cannot be disrupted, have the same disruption probabilities, the authors propose a Lagrangian relaxation algorithm to solve the problem. By varying the weights on the two objectives, the authors generate a tradeoff curve between the nominal cost and the failure cost and use it to demonstrate that substantial improvements in reliability (vs. the optimal classical solution) can often be obtained with a very small increase in nominal cost. Moreover, they show that the optimal number of facilities tends to be larger than that of the classical uncapacitated fixed-charge location problem (UFLP) in order to reduce the impact of each disruption, a phenomenon that is closely related to the risk-diversification effect (see Section 3.2).

One important shortcoming of Snyder and Daskin's (2005) model is the assumption that all facilities have the same disruption probability. When this assumption is relaxed, it becomes significantly more difficult to calculate the expected transportation cost for a given customer. Several approaches have been proposed to address this issue:

1. *Scenarios.* This approach entails enumerating all of (or a sample of) the disruption scenarios and formulating the problem as a stochastic programming model. This approach is intuitive and allows for statistical dependence among disruptions, but the problem size grows exponentially with the number of facilities. Models using this approach are proposed by Shen et al. (2011) and by Snyder et al. (2006).

2. *Non-Linear Probability Terms.* This approach uses nonlinear terms to calculate the probability that a customer is served by its rth closest facility. It is taken by Berman et al. (2007), Shen et al. (2011), Cui et al. (2010) and by Aboolian et al. (2012), discussed next, and by Zhan et al. (2008), discussed in Section 7.2.

3. *Reliable Backups.* Lim et al. (2010, 2012), discussed in Section 7.2, assume that each customer is assigned to (at most) one unreliable facility and then to a facility that may not fail, an approach that simplifies the formulation considerably but may be unrealistic in some settings (since, in practice, customers whose primary facility is disrupted may be served by other unreliable but non-disrupted facilities).

4. *Continuum Approximation (CA).* This approach uses a continuum approximation (Daganzo and

Newell 1986) in which customers are spread uniformly throughout some geographical area, and the parameters are expressed as a continuous function of the location. Cui et al. (2010) and Li and Ouyang (2010) take this approach and are discussed later in this section.

Next, we discuss papers that allow heterogeneous disruption probabilities and are referred to in the four approaches outlined above.

Berman et al. (2007) study a problem similar to Snyder and Daskin's (2005) RPMP. The resulting model, which they call the median problem with unreliable facilities (MPUF), uses highly non-linear terms to calculate the expected travel distance. Thus, the MPUF is less tractable than the RPMP but also more general. Structural results suggest that optimal solutions for the MPUF tend to be more centralized than those for the classical UFLP, and may even involve the co-location of two or more facilities at a single location.

Shen et al. (2011) present two formulations for the RFLP model of Snyder and Daskin (2005) with site-specific disruption probabilities. One uses a scenario approach within a stochastic programming framework, while the other involves computing the expected travel costs endogenously using highly nonlinear multiplicative terms in a manner similar to Berman et al. (2007). They propose a heuristic based on sample average approximation (Kleywegt et al. 2001) for the first model and two greedy-adding–type heuristics for the latter. Computational results suggest that one of the greedy-adding methods is superior to the other two heuristics.

Cui et al. (2010) also propose a nonlinear formulation for the problem with site-specific disruption probabilities, but, unlike the models by Berman et al. (2007) and Shen et al. (2011), which have terms consisting of multiple decision variables multiplied together, the Cui model only has products of a single continuous and a single discrete decision variable. The authors reformulate their model using the reformulation-linearization technique (RLT) of Sherali and Alameddine (1992), resulting in a model that is both linear in the decision variables and polynomial in the problem size. They also introduce a CA model that requires simulation and regression but may be solved in closed form to allow for managerial insights.

Aboolian et al. (2012) start with the same non-linear MIP model as Cui et al. (2010) but, rather than reformulating it using RLT, they formulate an approximate model in which the probability that a customer is assigned to a given facility is calculated assuming that all closer facilities are open. They prove that this approximate model provides a lower bound on the optimal objective value. The authors propose an algorithm in which: (1) the approximate model is solved; (2) the solution to the approximate model is used to generate a feasible solution, which is then improved using local-search techniques; (3) a cut is added to the approximate model that eliminates feasible solutions that have already been found; and the process repeats. Computational results show that this algorithm outperforms that of Cui et al. (2010), especially when the maximum number of assignments for a given customer is large or unlimited.

Li and Ouyang (2010) study another CA model, in which the disruptions are assumed to be geographically correlated. They introduce closed-form approximations for the total cost and the individual cost components and demonstrate that these are quite accurate.

Berman et al. (2009) assume that customers do not know which facilities are disrupted and must travel from facility to facility until they find a non-disrupted one. The authors show that the centralization and co-location effects noted for the MPUF are even stronger for this model, which they call the MPUF with incomplete information (MPUF-II). They suggest heuristics for the MPUF-II and examine the cost that results from the lack of disruption information.

Berman et al. (2010) consider a continuous version of their MPUF and MPUF-II models in which two unreliable facilities, with correlated disruptions, are to be located on a line segment. They develop closed-form solutions for the problems and perform parametric analysis to describe how the optimal locations and costs change when the disruption characteristics change.

Peng et al. (2011) study disruptions in the context of a network design problem in which location decisions must be made for unreliable nodes in a network. Disruptions are modeled using scenarios, for which no probabilities are assumed. The model minimizes the nominal cost (the cost of the scenario in which no disruptions occur) subject to a constraint requiring the relative regret in each scenario to be no more than a fixed constant p, an approach known as p-robustness (Kouvelis et al. 1992, Snyder and Daskin 2006). The authors propose a genetic algorithm–based metaheuristic.Another model for network design with scenario-based disruptions is formulated by Snyder et al. (2006); their model uses an expected cost objective and no p-robustness constraints. To the best of our knowledge, these and the paper discussed next are the only papers to consider disruptions in the context of a network design problem, loosely defined as a problem in which open/close decisions must be made at more than one echelon of a network.

Bundschuh et al. (2003) propose several models for designing robust and reliable supply chains. Under their definition, the "reliability" of a supply chain refers to the probability that all suppliers are operable, while "robustness" refers to the ability of the supply chain to maintain a given level of output after a disruption. (These definitions are somewhat at odds with one another since adding new suppliers improves robustness [through extra redundancy] but degrades reliability [since the probability that one or more suppliers fails increases with the number of suppliers].) The authors propose several IP models that add robustness or reliability considerations to the capacitated multi-echelon network design problem.

7.2 Interdiction, Location, and Fortification

Nearly all of the location models discussed above assume that disruptions occur probabilistically; the exception is Drezner's (1987) (p, q)-center problem, which, as noted above, protects against the worst-case cost that may occur if up to q facilities are disrupted. Models taking this minimax approach are similar in spirit to network interdiction problems (see, e.g., Wollmer 1964, Fulkerson and Harding 1977, Golden 1978, Janjarassuk and Linderoth 2008), which can be thought of as Stackelberg games with a leader and a follower. The leader tries to disrupt the network by interdicting it, i.e., destroying portions of it, and the follower then attempts, for example, to maximize flow or minimize the shortest path through the resulting network. Like many Stackelberg games, network interdiction problems are often modeled using bilevel optimization, consisting of an inner problem and an outer problem.

Another way to think about the difference between probabilistic and worst-case disruption models is that, in the former models, disruptions are considered to be exogenous, the result of weather, power failures, and so on, while in the latter they are endogenous, in the sense that disruptions are caused by actors that are modeled explicitly. Typically, endogenous disruptions may be affected by the decision-maker's actions and therefore are usually explicitly modeled by decision variables within the optimization problem, whereas exogenous disruptions cannot be affected and are not optimized but are modeled using stochastic processes.

In this section, we focus on interdiction models in the context of facility location. These differ from network interdiction problems in two main ways: (1) interdictions can only occur on the nodes, rather than on the arcs, of the network, and (2) the post-interdiction measure of interest is the distance or travel cost from customers to their assigned facilities, rather than, say, flows or shortest paths. In the subsections below, we first discuss models of interdiction, with no location component, then models that locate or fortify facilities to protect against future interdictions.

7.2.1 Interdiction

Church et al. (2004) study two interdiction problems in the context of two classical facility location objectives: p-median and max-covering. In particular, their models assume that the set of p facilities has already been constructed and choose r of them to interdict in order to cause as much deterioration as possible. The distance-based problem is called the r-interdiction median (RIM) problem, while the coverage-based one is called the r-interdiction covering (RIC) problem. Results suggest that interdiction may cause significant increases in cost, even for small r. Losada et al. (2010a) study a finite-horizon version of the RIM in which the r interdictions occur throughout the horizon and the facilities remain unavailable for a deterministic recovery time.

Church and Scaparra (2007) propose plotting, for a fixed set of p facilities and for each $r \in \{0, \dots, p\}$, the best- and worst-case loss that can result from an r-facility interdiction. They introduce the notion of the "reliability envelope," which is the region enclosed by the curve that connects the best-case points and the curve that connects the worst-case points. The shape and thickness of the envelope provides planners with information about which facilities to protect, which to shut down if planned disruptions are required, how much improvement to expect from fortification measures, etc.

Losada et al. (2012) extend this latter model to consider the case in which the interdictor can influence the probability of an attack's success. In particular, the interdictor may choose from a discrete set of attack levels, each with its own cost and probability of success, and must satisfy a budget constraint. As in many of the models cited above, calculating the probability that a given customer is assigned to a given facility is difficult. The closest-assignment rule further complicates the assignment-probability calculation. The authors propose three formulations to address these issues, ultimately settling on a network formulation in which the assignment probabilities are represented as flow variables.

7.2.2 Fortification/Design for Interdiction

The model by Church and Scaparra (2006) chooses q facilities to "fortify" (i.e., make non-disruptable), out of p existing facilities, in order to minimize the increase in demand-weighted distance caused by a worst-case r-facility interdiction. The model is known as the r-interdiction median problem with fortification (RIMF). This problem has a tri-level, min-max-min flavor: In the innermost problem, we seek to minimize the demand-weighted distance from customers to non-interdicted facilities; the interdictor attempts to maximize our minimum distance through his interdictions; and we attempt to minimize this maximum damage through our fortifications.

Scaparra and Church (2008*b*) reformulate the RIMF as a maximal-covering problem with precedence constraints and suggest lower and upper bounds that can be used to reduce the problem size. Scaparra and Church (2008*a*) propose an alternate solution method, this one based on a tree search procedure for the original bilevel programming problem. Aksen et al. (2009) extend the RIMF to consider budget constraints on the fortification resources and to assume that a linear capacity-expansion cost is incurred when customers are re-assigned to non-interdicted facilities.

Scaparra and Church (2012) propose a capacitated version of the RIMF, as do Liberatore et al. (2012); the latter paper also allows for partial disruptions and for disruptions that affect geographic regions, rather than individual facilities. Both papers reduce the "max-min" (interdictor–recourse) part of the problem to a single-level problem and then use the algorithm of Scaparra and Church (2008*a*) to solve the resulting overall problem.

Liberatore et al. (2011) extend Church and Scaparra's (2006) model to consider a stochastic number r of interdicted facilities; once r is determined, the attacker is assumed to perform the worst-case r-facility interdiction. The authors demonstrate numerically that failing to account for the uncertainty in the number of interdicted facilities can result in substantial error but that mis-estimating the scenario probabilities is unlikely to do so. Liberatore and Scaparra (2011) extend this model to consider regret-based objectives and show how to modify the problem-reduction techniques to apply to the new problem.

Along similar lines as Church and Scaparra (2006), O'Hanley and Church (2011) introduce a location–interdiction model based on the RIC model of Church et al. (2004). The objective is to choose p facilities to maximize a weighted sum of the non-disrupted demand-weighted coverage and the minimum demand-weighted coverage that occurs after a worst-case interdiction of r facilities. The key decision in O'Hanley and Church's model is the locations of the p facilities, whereas Church and Scaparra's (2006) model takes these as given and chooses which facilities to fortify.

O'Hanley et al. (2007*a*) discuss the location of nature reserves, which then may be disrupted or destroyed in the future by human encroachment or natural disasters. Each species is assumed to be covered by a given set of locations, and, unlike most location–disruption models, there is no opportunity to re-assign species (analogous to customers) to alternate sites following a disruption. The authors present two models. The first maximizes the expected species coverage after disruptions and the second model maximizes a weighted sum of the coverage with no disruptions and the worst-case coverage level after a disruption whose probability exceeds a specified threshold.

It is worth also noting another reserve-selection paper by O'Hanley et al. (2007*b*), which considers what one might call "reverse disruptions." In particular, the models recognize that the sites not selected as reserves may nevertheless continue to support species. That is, while O'Hanley et al. (2007*a*) (and most other location–disruption models) assume that some *chosen* sites may randomly *fail* to function, this paper assumes that some *non-chosen* sites may randomly *function*. The models themselves are similar to those by O'Hanley et al. (2007*a*).

Losada et al. (2010*b*) discuss fortification of facilities to protect against worst-case p-median losses of the type discussed by Losada et al. (2010*a*). The objective is to allocate a fixed fortification budget among the p existing facilities. The authors formulate a bilevel MIP and compare two decomposition methods: Benders decomposition, and a method based on super-valid inequalities (SVI). They find that the two methods perform roughly the same on average, but the Benders approach tends to perform better than the SVI approach for smaller instances and worse for larger ones.

Finally, we briefly discuss three papers that also consider fortification but in contexts other than facility location. Bailey et al. (2006) introduce a novel framework called SPAR (stochastic programming with adversarial recourse) for design-plus-interdiction problems. The model makes design decisions in the first stage of a stochastic program in order to minimize the sum of the design costs plus the expected damage from an adversary. The adversary's problem is modeled as a Markov decision process (MDP). The framework is general and can be used to model location–interdiction problems, although it is not presented explicitly in this context. The model by Scaparra and Cappanera (2011) determines which nodes in a network to fortify in order to minimize the shortest path between two specified nodes after a worst-case interdiction that disrupts components of the network. Golany et al. (2009) determine the optimal locations to fortify when defense resources are constrained; the disruption cost is measured by a scalar that is multiplied by the level of damage at each site. They discuss the differences between the optimal fortification strategies under probabilistic risk (caused by natural events) and strategic risk (caused by malicious agents).

7.2.3 Fortification During Design Phase

The papers discussed in this subsection assume that the facility locator has some control over the disruption probability. Therefore, these models assume that fortification decisions can be made during the design phase, whereas the papers discussed in the previous subsection assume that facilities are fortified after they are built. The two approaches are similar but lead to different modeling approaches.

Zhan et al. (2008) propose an extension of Snyder and Daskin's (2005) RFLP model in which the disruption probability is chosen endogenously, and the fixed cost depends on the probability chosen. The authors calculate the probability that a given facility serves a given customer using highly non-linear terms, as in Shen et al. (2011) (see Section 7.1), and they propose two greedy-adding heuristics and one genetic algorithm to solve the problem approximately.

Like Church and Scaparra (2006) and O'Hanley and Church (2011), Lim et al. (2010) ask where to locate or fortify facilities in order to protect against attacks, but the attacks they consider are random, not worst-case. Their model chooses locations for both unreliable and reliable (i.e., fortified) facilities; the reliable facilities have a higher fixed cost but cannot be disrupted. The objective is to minimize the fixed cost plus the expected transportation cost. They prove that there exist thresholds such that if the disruption probabilities for every facility are above or below the thresholds, then the problem reduces to the classical UFLP.

In contrast to the discrete location models discussed above, Lim et al. (2012) use a continuous approximation model in which customers are spread uniformly in a geometric region and the objective is to find the optimal number of locations. Like the model by Lim et al. (2010), this model locates both unreliable and reliable facilities to minimize the fixed cost plus expected transportation cost. They prove that there exists a threshold, which depends only on the fixed costs, such that, if the disruption probability q is greater than the threshold, then it is optimal to open only reliable facilities, and otherwise it is optimal to open both types of facilities.

7.3 Location–Inventory Models

In this section we discuss four papers that integrate inventory management into the facility location decision. The models can be thought of as combining location–inventory models (Daskin et al. 2002, Shen et al. 2003) with the reliable location models of the type discussed in Section 7.1. Qi and Shen (2007) assumes that the facilities are unreliable and the customers they serve use routine sourcing (i.e., ordering proactively from multiple facilities) to mitigate the supply uncertainty. The customers must place orders before the state of the facilities is known. Each customer acts as a newsvendor with multiple unreliable suppliers (in this case, the facilities), as in Dada et al. (2007). The model determines the optimal facility locations to maximize the profit, accounting for the fixed cost, the (deterministic) inventory cost at the facilities, the expected inventory cost at the customers (including holding and stockout costs), and the transportation cost.

In Qi et al. (2010), the facilities and their suppliers can both be disrupted, and when disruptions occur, customers may not be reassigned to other facilities; therefore, inventory is the only available mitigation strategy. The disruption process and the facilities' inventory problem are identical to those suggested by Qi et al. (2009) (see Section 6.1), and the approximation from that paper is used by Qi et al. (2010) to calculate the inventory cost in closed form in the objective function of the location model. The authors demonstrate numerically that significant cost savings can be achieved by considering disruptions during the facility-location phase.

Jeon et al. (2008) allow customers to be reassigned after disruptions occur (as in the models in Section 7.1) and account for the expected inventory cost after reassignment; this model thus uses contingent rerouting as its mitigation strategy and uses inventory as a tool for coping with demand uncertainty only. Their objective function includes the fixed cost of locating facilities and the expected transportation and inventory costs. Since rerouting is the strategy for coping with disruptions, the inventory costs do not explicitly account for disruptions. However, they do account for disruptions implicitly since the demand served by each facility, and the resulting inventory cost, is a function of the disruptions. This cost is difficult to calculate, so the authors propose a closed-form approximation and demonstrate its accuracy numerically. Whereas the location–inventory model tends to open fewer facilities than the classical UFLP (because of economies of scale and the risk-pooling effect) and the reliable location model tends to open more facilities (because of the risk-diversification effect), this hybrid model balances these competing tendencies. Mir-Bahador and Jabbarzadeh (2009) formulate a similar model and propose a genetic algorithm to solve it.

Mak and Shen (2012) assume that customers may be reassigned to non-disrupted facilities but that those facilities must place replenishment orders before disruptions are realized; therefore, the model uses both contingent rerouting and inventory (at the reassigned facilities) as mitigation strategies. The replenishment order at a non-disrupted facility is used to serve that facility's usual customers as well as customers that are reassigned to it, and therefore inventory is used both to cope with demand uncertainty and to mitigate disruptions at other facilities. There is no fixed ordering cost, so each facility follows a base-stock policy. The authors consider both Bernoulli (time-independent) disruptions and Markovian (time-dependent) disruptions and suggest Lagrangian decomposition (variable splitting) algorithms for both.

8 Discussion

In this paper, we have reviewed nearly 150 scholarly works from the OR/MS literature that deal with the topic of supply disruptions. This literature has been growing quickly in the past few years, motivated by recent high-profile disruptions, increased globalization, and the vulnerabilities introduced by lean operations. We have attempted to provide as comprehensive a review of this literature as possible, to serve as a reference for researchers currently working in the field, and as a starting point for those who wish to begin to explore it.

The literature on supply disruptions is likely to continue to grow over the coming years. We have identified the following topics that we believe are particularly promising and important as avenues for future research:

1. *Risk aversion.* Relatively few of the models discussed above consider risk aversion; nearly all assume a risk-neutral decision maker who wishes to optimize the expected value of the objective function. Since many types of disruptions are rare, decision makers may tend to be risk averse and may wish to optimize other measures, such as worst-case objectives, p-robustness, and conditional value-at-risk (CVaR). Robust optimization is a promising tool for such problems.

2. *Parameter estimation.* Since historical data on disruptions may be scant, and since new types of disruptions are constantly developing, it can be very difficult to estimate parameters such as the disruption and recovery probabilities/rates. However, nearly all of the models cited above assume that such parameters are known with certainty. Therefore, it will be critical to develop techniques for better estimating the disruption parameters or, failing that, to develop models that perform well even if the parameters are unknown. Here, too, robust optimization may play an important role, but in this case the uncertainty is even more pronounced, since the parameters of the stochastic processes are themselves unknown.

3. *Integration of proactive and reactive strategies.* Most models assume a single type of mitigation strategy. However, in practice, it can be quite effective to use multiple strategies, and especially to develop both proactive and reactive mitigation strategies simultaneously. Future research may determine the value and optimal deployment of such multi-pronged mitigation approaches.

4. *Improved understanding of multi-echelon systems.* Disruptions are multi-echelon events, with upstream disruptions propagating downstream in the form of stockouts. However, the literature on multi-echelon systems under the risk of disruptions is limited. There is a need for new models that improve our understanding of how disruptions propagate and how to mitigate disruptions in multi-echelon systems.

5. *Behavioral studies.* Since disruptions are abnormal events, managers may tend to deviate from optimal solutions prescribed by the models discussed throughout this paper. Behavioral studies will be important to model this irrational behavior. Research questions may include how to describe or explain managers' behavior under disruptions, as well as how a rational manager in a multi-player supply chain can account for this behavior when solving his or her own optimization problem.

6. *Endogenous disruption processes.* Although endogenous disruptions have been discussed in the context of facility location (see Section 7.2), most other areas of the disruption literature assume

disruptions are exogenous. However, many real-world disruptions are affected by the firm's actions or situation. For example, labor strikes are often timed to occur when demand is high (e.g., in the lead-up to the Christmas holiday season, as in the case of the west-coast port lockout of 2002) or inventory is low so that striking workers have more bargaining power. Moreover, reactive mitigation strategies under endogenous disruption processes may affect the recovery duration. For example, the firm may concede to the labor union in order to hasten the end of the disruption.

7. *Endogenous demand processes.* The demand for certain products may change depending on the state of the disruption process. For example, the demand for bottled water, fuel, and canned goods surges before, during, and after some disruptions. There is a need for models that account for non-stationary demand processes that depend on the disruption state. Such models will be related to the literature on disaster relief and public-sector supply chains.

References

Aboolian, R., Cui, T. and Shen, Z.-J. M. (2012), 'An efficient approach for solving reliable facility location models', *INFORMS Journal of Computing* . Forthcoming.

Aksen, D., Piyade, N. and Aras, N. (2009), 'The budget constrained r-interdiction median problem with capacity expansion', *Central European Journal of Opearations Research* **18**(3), 269–291.

Anupindi, R. and Akella, R. (1993), 'Diversification under supply uncertainty', *Management Science* **39**(8), 944–963.

Argon, N. T., Güllü, R. and Erkip, N. (2001), 'Analysis of an inventory system under backorder correlated deterministic demand and geometric supply process', *International Journal of Production Economics* **71**(1-3), 247–254.

Arreola-Risa, A. and DeCroix, G. A. (1998), 'Inventory management under random supply disruption and partial backorders', *Naval Research Logistics* **45**, 687–703.

Atan, Z. and Snyder, L. (2010), Inventory strategies to manage supply disruptions, *in* H. Gurnani, A. Mehrotra and S. Ray, eds, 'Managing Supply Disruptions', Springer-Verlag, pp. 115–139.

Atan, Z. and Snyder, L. V. (2012), Disruptions in one-warehouse multiple-retailer systems, Working paper, Lehigh University.

Babich, V. (2006), 'Vulnerable options in supply chains: Effect of the supplier competition', *Naval Research Logistics* **53**(7), 656–673.

Babich, V. (2010), 'Independence of capacity ordering and financial subsidies to risky suppliers', *Manufacturing & Service Operations Management* **12**(4), 583–607.

Babich, V., Burnetas, A. N. and Ritchken, P. H. (2007), 'Competition and diversification effects in supply chains with supplier default risk', *Manufacturing and Service Operations Management* **9**(2), 123–146.

Bailey, M. D., Shechter, S. M. and Schaefer, A. J. (2006), 'SPAR: stochastic programming with adversarial recourse', *Operations Research Letters* **34**(3), 307–315.

Bar-Lev, S. K., Parlar, M. and Perry, D. (1993), 'Impulse control of a Brownian inventory system with supplier uncertainty', *Journal of Stochastic Analysis and Applications* **11**, 11–27.

Berger, P. D., Gerstenfeld, A. and Zeng, A. Z. (2004), 'How many suppliers are best? a decision-analysis approach', *Omega* **32**, 9–15.

Berger, P. D. and Zeng, A. Z. (2006), 'Single versus multiple sourcing in the presence of risks', *The Journal of the Operational Research Society* **57**(3), 250–261.

Berk, E. and Arreola-Risa, A. (1994), 'Note on "Future supply uncertainty in EOQ models"', *Naval Research Logistics* **41**, 129–132.

Berman, O., Krass, D. and Menezes, M. B. C. (2007), 'Facility reliability issues in network p-median problems: Strategic centralization and co-location effects', *Operations Research* **55**(2), 332–350.

Berman, O., Krass, D. and Menezes, M. B. C. (2009), 'Locating facilities in the presence of disruptions and incomplete information', *Decision Sciences* **40**(4), 845–868.

Berman, O., Krass, D. and Menezes, M. B. C. (2010), Location problems with two unreliable facilities on a line allowing correlated failures. Working Paper, Rotman School of Management, University of Toronto, Toronto, Ontario, Canada.

Bielecki, T. and Kumar, P. R. (1988), 'Optimality of zero-inventory policies for unreliable manufacturing systems', *Operations Research* **36**(4), 532–541.

Bundschuh, M., Klabjan, D. and Thurston, D. L. (2003), Modeling robust and reliable supply chains, Submitted manuscript, University of Illinois at Urbana-Champaign.

Chao, H.-P. (1987), 'Inventory policy in the presence of market disruptions', *Operations Research* **35**(2), 274–281.

Chao, H.-P., Chapel, S. W., Clark, C. E., J., Morris, P. A., Sandling, M. J. and Grimes, R. C. (1989), 'EPRI reduces fuel inventory costs in the electric utility industry', *Interfaces* **19**(1), 48–67.

Chaturvedi, A. and Martínez-de Albéniz, V. (2011), 'Optimal procurement design in the presence of supply risk', *Manufacturing and Service Operations Research* **59**(1), 109–124.

Chopra, S., Reinhardt, G. and Mohan, U. (2007), 'The importance of decoupling recurrent and disruption risks in a supply chain', *Naval Research Logistics* **54**(5), 544–555.

Chopra, S. and Sodhi, M. S. (2004), 'Managing risk to avoid supply-chain breakdown', *Sloan Management Review* **46**(1), 53–61.

Church, R. L. and Scaparra, M. P. (2006), 'Protecting critical assets: The r-interdiction median problem with fortification', *Geographical Analysis* **39**(2), 129–146.

Church, R. L. and Scaparra, M. P. (2007), Analysis of facility systems' reliability when subject to attack or a natural disaster, *in* A. T. Murray and T. H. Grubesic, eds, 'Reliability and Vulnerability in Critical Infrastructure: A QuantitativeGeographic Perspective', Springer, chapter 11, pp. 221–241.

Church, R. L., Scaparra, M. P. and Middleton, R. S. (2004), 'Identifying critical infrastructure: The median and covering facility interdiction problems', *Annals of the Association of American Geographers* **94**(3), 491–502.

Clark, A. J. and Scarf, H. (1960), 'Optimal policies for a multi-echelon inventory problem', *Management Science* **6**(4), 475–490.

Clausen, J., Hansen, J., Larsen, J. and Larsen, A. (2001), 'Disruption management', *OR/MS Today* **28**(5).

Colbourn, C. (1987), *The Combinatorics of Network Reliability*, Oxford University Press, New York.

Cui, T., Ouyang, Y. and Shen, Z.-J. M. (2010), 'Reliable facility location design under the risk of disruptions', *Operations Research* **58**(4), 998–1011.

Dada, M., Petruzzi, N. and Schwarz, L. (2007), 'A newsvendor's procurement problem when suppliers are unreliable', *Manufacturing and Service Operations Management* **9**(1), 9–32.

Daganzo, C. F. and Newell, G. F. (1986), 'Configuration of physical distribution networks', *Networks* **16**, 113–132.

Daskin, M. S. (1995), *Network and Discrete Location: Models, Algorithms, and Applications*, Wiley, New York.

Daskin, M. S., Coullard, C. R. and Shen, Z.-J. M. (2002), 'An inventory-location model: Formulation, solution algorithm and computational results', *Annals of Operations Research* **110**, 83–106.

DeCroix, G. A. (2012), 'Inventory management for an assembly system subject to supply disruptions', *Management Science* . Forthcoming.

Deleris, L. A. and Erhun, F. (2005), Risk management in supply networks using Monte-Carlo simulation, *in* M. E. Kuhl, N. M. Steiger, F. B. Armstrong and J. A. Joines, eds, 'Proceedings of the 2005 Winter Simulation Conference'.

Drezner, Z. (1987), 'Heuristic solution methods for two location problems with unreliable facilities', *Journal of the Operational Research Society* **38**(6), 509–514.

Drezner, Z., ed. (1995), *Facility Location: A Survey of Applications and Methods*, Springer-Verlag, New York.

Drezner, Z. and Hamacher, H. W., eds (2002), *Facility Location: Applications and Theory*, Springer-Verlag, New York.

Ellis, S. C., Henry, R. M. and Shockley, J. (2010), 'Buyer perceptions of supply disruption risk: A behavioral view and empirical assessment', *Journal of Operations Management* **28**, 34–46.

Eppen, G. D. (1979), 'Effects of centralization on expected costs in a multi-location newsboy problem', *Management Science* **25**(5), 498–501.

Federgruen, A. and Yang, N. (2008), 'Selecting a portfolio of suppliers under demand and supply risks', *Operations Research* **56**(4), 916–936.

Federgruen, A. and Yang, N. (2009), 'Optimal supply diversification under general supply risks', *Operations Research* **57**(6), 1451–1468.

Fulkerson, D. R. and Harding, G. C. (1977), 'Maximizing the minimum source-sink path subject to a budget constraint', *Mathematical Programming* **13**, 116–118.

Gao, L. and Li, Z. (2010), Managing supply interruptions with contract coordination and information sharing. Working Paper, Anderson Graduate School of Management, University of California, Riverside, CA.

Golany, B., Kaplan, E. H., Marmur, A. and Rothblum, U. G. (2009), 'Nature plays with dice—terrorists do not: Allocating resources to counter strategic versus probabilistic risks', *European Journal of Operational Research* **192**(1), 198–208.

Golden, B. (1978), 'A problem in network interdiction', *Naval Research Logistics Quarterly* **25**, 711–713.

Groenevelt, H., Pintelon, L. and Seidmann, A. (1992*a*), 'Production batching with machine breakdowns and safety stocks', *Operations Research* **40**(5), 959–971.

Groenevelt, H., Pintelon, L. and Seidmann, A. (1992*b*), 'Production lot sizing with machine breakdowns', *Management Science* **38**(1), 104–123.

Grosfeld-Nir, A. and Gerchak, Y. (2004), 'Multiple lotsizing in production to order with random yields: Review of recent advances', *Annals of Operations Research* **126**(1-4), 43–69.

Güllü, R., Önol, E. and Erkip, N. (1997), 'Analysis of a deterministic demand production/inventory system under nonstationary supply uncertainty', *IIE Transactions* **29**, 703–709.

Güllü, R., Önol, E. and Erkip, N. (1999), 'Analysis of an inventory system under supply uncertainty', *International Journal of Production Economics* **59**, 377–385.

Gupta, D. (1996), 'The (Q, r) inventory system with an unreliable supplier', *INFOR* **34**(2), 59–76.

Gürler, U. and Parlar, M. (1997), 'An inventory problem with two randomly available suppliers.', *Operations Research* **45**(6), 904–918.

Gurnani, H. and Shi, M. (2006), 'A bargaining model for a first-time interaction under asymmetric beliefs of supply reliability', *Management Science* **52**(6), 865–880.

Haksoz, C. and Kadam, A. (2008), 'Supply risk in fragile contracts', *MIT Sloan Management Review* **49**(52), 7–8.

Heimann, D. and Waage, F. (2007), 'A closed-form approximation solution for an inventory model with supply disruptions and non-ZIO reorder policy', *Journal of Systemics, Cybernetics, and Informatics* **5**(4), 1–12.

Hendricks, K. B. and Singhal, V. R. (2003), 'The effect of supply chain glitches on shareholder wealth', *Journal of Operations Management* **21**(5), 501–522.

Hendricks, K. B. and Singhal, V. R. (2005*a*), 'Association between supply chain glitches and operating performance', *Management Science* **51**(5), 695–711.

Hendricks, K. B. and Singhal, V. R. (2005*b*), 'An empirical analysis of the effect of supply chain disruptions on long-run stock price performance and equity risk of the firm', *Production and Operations Management* **14**(1), 35–52.

Hopp, W. J., Iravani, S. M. R. and Liu, Z. (2009), Strategic risk from supply chain disruptions. Working Paper, Stephen M. Ross School of Business, University of Michigan, Ann Arbor, MI.

Hopp, W. J. and Liu, Z. (2006), Protecting supply chain networks against catastrophic failures. Working Paper, Department of Industrial Engineering and Management Science, Northwestern University, Evanston, IL.

Janjarassuk, U. and Linderoth, J. T. (2008), 'Reformulation and sampling to solve a stochastic network interdiction problem', *Networks* **52**(3), 120–132.

Jeon, H.-M., Snyder, L. V. and Shen, Z.-J. M. (2008), A location-inventory model with supply disruptions, Working paper, Lehigh University.

Jordan, W. C. and Graves, S. C. (1995), 'Principles on the benefits of manufacturing process flexibility', *Management Science* **41**(4), 577–594.

Journal of Commerce (2005), 'Picking up the pieces: shippers and carriers seek alternatives to transportation networks damaged by Hurricane Katrina', *Journal of Commerce* .

Kalpakam, S. and Sapna, K. P. (1997), 'A lost sales inventory system with supply uncertainty', *Computers & Mathematics with Applications* **33**(3), 81–93.

Kerivin, H. and Mahjoub, A. R. (2005), 'Design of survivable networks: A survey', *Networks* **46**(1), 1–21.

Kleindorfer, P. R. and Saad, G. H. (2005), 'Managing disruption risks in supply chains', *Production and Operations Management* **14**(1), 53–68.

Kleiss, K. (2009), 'Edmonton's striking Safeway workers brace for a long strike', *Calgary Herald* .

Kleywegt, A. J., Shapiro, A. and Homem-de Mello, T. (2001), 'The sample average approximation method for stochastic discrete optimization', *SIAM Journal on Optimization* **12**(2), 479–502.

Kouvelis, P., Kurawarwala, A. A. and Gutiérrez, G. J. (1992), 'Algorithms for robust single and multiple period layout planning for manufacturing systems', *European Journal of Operational Research* **63**, 287–303.

Lee, S.-D. (2001), 'On solving unreliable planar location problems', *Computers and Operations Research* **28**(4), 329–344.

Lewis, B. M., Erera, A. L. and White, C. C. (2005), An inventory control model with possible border disruptions, Working paper, Georgia Institute of Technology, Atlanta, GA.

Li, G., Lin, Y., Wang, S. and Yan, H. (2006), 'Enhancing agility by timely sharing of supply information', *Supply Chain Management: An International Journal* **11**(5), 425–435.

Li, J., Wang, S. and Cheng, T. C. E. (2010), 'Competition and cooperation in a single-retailer two-supplier supply chain with supply disruption', *International Journal of Production Economics* **124**, 137–150.

Li, X. and Ouyang, Y. (2010), 'A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions', *Transportation Research–Part B* **44**(4), 535–548.

Li, Z., Xu, S. H. and Hayya, J. (2004), 'A periodic-review inventory system with supply interruptions', *Probability in the Engineering an Informational Sciences* **18**, 33–53.

Liberatore, F. and Scaparra, M. P. (2011), 'Optimizing protection strategies for supply chains: Comparing classic decision making criteria in an uncertain environment', *Annals of the Association of American Geographers* **101**(6), 1241–1258.

Liberatore, F., Scaparra, M. P. and Daskin, M. S. (2011), 'Analysis of facility protection strategies against an uncertain number of attacks: The stochastic R-interdiction median problem with fortification', *Computers & Operations Research* **38**(1), 357–366.

Liberatore, F., Scaparra, M. P. and Daskin, M. S. (2012), 'Optimization methods for hedging against disruptions with ripple effects in location analysis', *Omega* **40**(1), 21–30.

Lim, M., Bassamboo, A., Chopra, S. and Daskin, M. S. (2011), Flexibility and fragility: Supply chain network design with disruption risks. Working Paper, Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, IL.

Lim, M., Daskin, M. S., Bassamboo, A. and Chopra, S. (2010), 'A facility reliability problem: Formulation, properties, and algorithm', *Naval Research Logistics* **57**(1), 58–70.

Lim, M., Daskin, M. S., Bassamboo, A. and Chopra, S. (2012), 'Facility location decisions with random disruptions and imperfect estimation', *Manufacturing and Service Operations Management* . Forthcoming.

Liu, B. and Cao, J. (1999), 'Analysis of a production-inventory system with machine breakdowns and shutdowns', *Computers and Operations Research* **26**, 73–91.

Losada, C., Scaparra, M. P. and Church, R. L. (2010*a*), Interdiction of p-median systems with facility recovery time and frequent disruptions: A resiliency analysis. Working paper, Kent Business School, University of Kent, UK.

Losada, C., Scaparra, M. P. and Church, R. L. (2010*b*), 'On a bi-level formulation to protect uncapacitated p-median systems with facility recovery time and frequent disruptions', *Electronic Notes in Discrete Mathematics* **36**, 591–598.

Losada, C., Scaparra, M. P., Church, R. L. and Daskin, M. S. (2012), 'The stochastic interdiction median problem with disruption intensity levels', *Annals of Operations Research* . Forthcoming.

Mak, H.-Y. and Shen, Z.-J. M. (2012), 'Risk-diversification and risk-pooling in supply chain network design', *IIE Transactions* **44**(8), 603–621.

Meyer, R. R., Rothkopf, M. H. and Smith, S. A. (1979), 'Reliability and inventory in a production-storage system', *Management Science* **25**(8), 799–807.

Mir-Bahador, A. and Jabbarzadeh, A. (2009), An integrated model for location-inventory problem with random disruptions, *in* 'Proceedings of the International Conference on Computers & Industrial Engineering (CIE39)', pp. 687–703.

Mohebbi, E. (2003), 'Supply interruptions in a lost-sales inventory system with random lead time', *Computers and Operations Research* **30**, 411–426.

Mohebbi, E. (2004), 'A replenishment model for the supply-uncertainty problem', *International Journal of Production Economics* **87**(1), 25–37.

Mohebbi, E. and Hao, D. (2006), 'When supplier's availability affects the replenishment lead time: An extension of the supply-interruption problem', *European Journal of Operational Research* **175**, 992–1008.

Mohebbi, E. and Hao, D. (2008), 'An inventory model with non-resuming randomly interruptible lead time', *International Journal of Production Economics* **114**, 755–768.

Moinzadeh, K. and Aggarwal, P. (1997), 'Analysis of a production / inventory system subject to random disruptions', *Management Science* **43**(11), 1577–1588.

Murray, A. T. and Grubesic, T. H., eds (2006), *Reliability and Vulnerability in Critical Infrastructure: A Quantitative Geographic Perspective*, Springer.

Natarajarathinam, M., Capar, I. and Narayanan, A. (2009), 'Managing supply chains in times of crisis: a review of literature and insights', *International Journal of Physical Distribution & Logistics Management* **39**(7), 535–573.

O'Hanley, J. R. and Church, R. L. (2011), 'Designing robust coverage networks to hedge against worst-case facility losses', *European Journal of Operational Research* **209**(1), 23–36.

O'Hanley, J. R., Church, R. L. and Gilless, J. K. (2007*a*), 'The importance of *in situ* site loss in nature reserve selection: Balancing notions of complementarity and robustness', *Biological Conservation* **135**(2), 170–180.

O'Hanley, J. R., Church, R. L. and Gilless, J. K. (2007*b*), 'Locating and protecting critical reserve sites to minimize expected and worst-case losses', *Biological Conservation* **134**(1), 130–141.

Özekici, S. and Parlar, M. (1999), 'Inventory models with unreliable suppliers in a random environment', *Annals of Operations Research* **91**, 123–136.

Pang, G. and Whitt, W. (2009), 'Service interruptions in large-scale service systems', *Management Science* **55**(9), 1499–1512.

Parlar, M. (1997), 'Continuous-review inventory problem with random supply interruptions', *European Journal of Operational Research* **99**, 366–385.

Parlar, M. (2000), 'Probabilistic analysis of renewal cycles: An application to a non-Markovian inventory problem with multiple objectives', *Operations Research* **48**(2), 243–255.

Parlar, M. and Berkin, D. (1991), 'Future supply uncertainty in EOQ models', *Naval Research Logistics* **38**, 107–121.

Parlar, M. and Perry, D. (1995), 'Analysis of a (q, r, t) inventory policy with deterministic and random yields when future supply is uncertain', *European Journal of Operational Research* **84**, 431–443.

Parlar, M. and Perry, D. (1996), 'Inventory models of future supply uncertainty with single and multiple suppliers', *Naval Research Logistics* **43**, 191–210.

Parlar, M., Wang, Y. and Gerchak, Y. (1995), 'A periodic review inventory model with Markovian supply availability', *International Journal of Production Economics* **42**, 131–136.

Peng, P., Snyder, L. V., Liu, Z. and Lim, A. (2011), 'Design of reliable logistics network with facility disruptions', *Transportation Research–Part B* **45**(8), 1190–1211.

Posner, M. J. and Berg, M. (1989), 'Analysis of a production-inventory system with unreliable production facility', *Operations Research Letters* **8**, 339–345.

Qi, L. (2009), A continuous-review inventory model with dual-sourcing strategy random disruptions at the primary supplier. Working paper.

Qi, L. and Shen, Z.-J. M. (2007), 'A supply chain design model with unreliable supply', *Naval Research Logistics* **54**(8), 829–844.

Qi, L., Shen, Z.-J. M. and Snyder, L. V. (2009), 'A continuous-review inventory model with disruptions at both supplier and retailer', *Production and Operations Management* **18**(5), 516–532.

Qi, L., Shen, Z.-J. M. and Snyder, L. V. (2010), 'The effect of supply disruptions on supply chain design decisions', *Transportation Science* **44**(2), 274–289.

Rong, Y., Shen, Z.-J. M. and Snyder, L. V. (2009*a*), 'The impact of ordering behavior on order-quantity variability: A study of forward and reverse bullwhip effects', *Flexible Services and Manufacturing Journal* **20**(1), 95–124.

Rong, Y., Shen, Z.-J. M. and Snyder, L. V. (2009*b*), Pricing during disruptions: A cause of the reverse bullwhip effect, Working paper, Lehigh University, Bethlehem, PA.

Rong, Y., Snyder, L. V. and Shen, Z.-J. M. (2009), Bullwhip and reverse bullwhip effects under rationing game, Working paper, Lehigh University, Bethlehem, PA.

Rosling, K. (1989), 'Optimal inventory policies for assembly systems under random demands', *Operations Research* **37**(4), 565–579.

Ross, A. M., Rong, Y. and Snyder, L. V. (2008), 'Supply disruptions with time-dependent parameters', *Computers and Operations Research* **35**(11), 3504–3529.

Ruiz-Torres, A. J. and Mahmoodi, F. (2007), 'The optimal number of suppliers considering the costs of individual supplier failures', *Omega* **35**, 104–115.

Saghafian, S. and Van Oyen, M. P. (2012), 'The value of flexible suppliers and disruption risk information: Newsvendor analysis with recourse', *IIE Transactions* **44**(10), 834–867.

Sawik, T. (2011), 'Selection of supply portfolio under disruption risks', *Omega* **39**, 194–208.

Scaparra, M. P. and Cappanera, P. (2011), 'Optimal allocation of protective resources in shortest-path networks', *Transportation Science* **45**(1), 64–80.

Scaparra, M. P. and Church, R. L. (2008*a*), 'A bilevel mixed-integer program for critical infrastructure protection planning', *Computers and Operations Research* **35**(6), 1905–1923.

Scaparra, M. P. and Church, R. L. (2008*b*), 'An exact solution approach for the interdiction median problem with fortification', *European Journal of Operational Research* **189**, 76–92.

Scaparra, M. P. and Church, R. L. (2012), 'Protecting supply systems to mitigate potential disaster: A model to fortify capacitated facilities', *International Regional Science Review* **35**(2), 188–210.

Schmitt, A. J. (2011), 'Strategies for customer service level protection under multi-echelon supply chain disruption risk', *Transportaion Research Part B: Methodological* **45**(8), 1266–1283.

Schmitt, A. J. and Singh, M. (2009), Quantifying supply chain disruption risk using Monte Carlo and discrete-event simulation, *in* M. D. Rossetti, R. R. Hill, B. Johansson, A. Dunkin and R. G. Ingalls, eds, 'Proceedings of the 2009 Winter Simulation Conference'.

Schmitt, A. J. and Singh, M. (2011), A quantitative analysis of disruption risk in a multi-echelon supply chain. Working Paper, Center for Transportation and Logistics, MIT.

Schmitt, A. J. and Snyder, L. V. (2012), 'Infinite-horizon models for inventory control under yield uncertainty and disruptions', *Computers and Operations Research* **39**(4), 850–862.

Schmitt, A. J., Snyder, L. V. and Shen, Z.-J. M. (2010), 'Inventory systems with stochastic demand and supply: Properties and approximations', *European Journal of Operational Research* **206**(2), 313–328.

Schmitt, A. J., Snyder, L. V. and Shen, Z.-J. M. (2011), Centralization versus decentralization: Risk pooling, risk diversification, and supply uncertainty in a one-warehouse multiple-retailer system. Working Paper, P.C. Rossin College of Engineering and Applied Sciences, Lehigh University, Bethlehem, PA.

Schmitt, A. J. and Tomlin, B. T. (2012), Sourcing strategies to manage supply disruptions, *in* H. Gurnani, A. Mehrotra and S. Ray, eds, 'Managing Supply Disruptions', Springer-Verlag, pp. 51–72.

Serel, D. (2008), 'Inventory and pricing decisions in a single-period problem involving risky supply', *International Journal of Production Economics* **116**(1), 115–128.

Shen, Z.-J. M., Coullard, C. R. and Daskin, M. S. (2003), 'A joint location-inventory model', *Transportation Science* **37**(1), 40–55.

Shen, Z.-J. M., Zhan, R. L. and Zhang, J. (2011), 'The reliable facility location problem: Formulations, heuristics, and approximation algorithms', *INFORMS Journal on Computing* **23**, 470–482.

Sherali, H. D. and Alameddine, A. (1992), 'A new reformulation-linearization technique for bilinear programming problems', *Journal of Global Optimization* **2**(4), 379–410.

Shier, D. (1991), *Network Reliability and Algebraic Structures*, Clarendon Press, Oxford.

Shooman, M. L. (2002), *Reliability of computer systems and networks: Fault tolerance, analysis,and design*, John Wiley & Sons, New York.

Shou, B., Huang, J. and Li, Z. (2011), 'Managing supply uncertainty under chain-b-chain competition'. Working paper, College of Business, City University of Hong Kong.

Simison, R. L. (1998), 'GM says strike reduced its earnings by $2.83 billion in 2nd and 3rd periods', *Wall Street Journal* p. 1.

Snyder, L. V. (2006), 'Facility location under uncertainty: A review', *IIE Transactions* **38**(7), 537–554.

Snyder, L. V. (2012), A tight approximation for a continuous-review inventory model with supplier disruptions. Working Paper, P.C. Rossin College of Engineering and Applied Sciences, Lehigh University, Bethlehem, PA.

Snyder, L. V. and Daskin, M. S. (2005), 'Reliability models for facility location: The expected failure cost case', *Transportation Science* **39**(3), 400–416.

Snyder, L. V. and Daskin, M. S. (2006), 'Stochastic p-robust location problems', *IIE Transactions* **38**(11), 971–985.

Snyder, L. V. and Daskin, M. S. (2007), Models for reliable supply chain network design, *in* A. T. Murray and T. H. Grubesic, eds, 'Reliability and Vulnerability in Critical Infrastructure: A Quantitative-Geographic Perspective', Springer, chapter 13, pp. 257–289.

Snyder, L. V., Scaparra, M. P., Daskin, M. L. and Church, R. C. (2006), Planning for disruptions in supply chain networks, *in* M. P. Johnson, B. Norman and N. Secomandi, eds, 'Tutorials in Operations Research', INFORMS, chapter 9, pp. 234–257.

Snyder, L. V. and Shen, Z.-J. M. (2006), Supply and demand uncertainty in multi-echelon supply chains. Working Paper, P.C. Rossin College of Engineering and Applied Sciences, Lehigh University, Bethlehem, PA.

Snyder, L. V. and Tomlin, B. T. (2008), Inventory management with advanced warning of disruptions. Working Paper, P.C. Rossin College of Engineering and Applied Sciences, Lehigh University, Bethlehem, PA.

Song, J.-S. and Zipkin, P. H. (1996), 'Inventory control with information about supply conditions', *Management Science* **42**(10), 1409–1419.

Swaminathan, J. M. and Shanthikumar, J. G. (1999), 'Supplier diversification: Effect of discrete demand', *Operations Research Letters* **24**, 213–221.

Swinney, R. and Netessine, S. (2009), 'Long-term contracts under the threat of supplier default', *Manufacturing and Service Operations Management* **11**(1), 109–127.

Tang, C. S. (2006*a*), 'Perspectives in supply chain risk management', *International Journal of Production Economics* **103**(2), 451–488.

Tang, C. S. (2006*b*), 'Robust strategies for mitigating supply chain disruptions', *International Journal of Logistics* **9**(1), 33–45.

Tang, C. S. and Tomlin, B. T. (2008), 'The power of flexibility for mitigating supply chain risks', *International Journal of Production Economics* **116**(1), 12–27.

Tomlin, B. T. (2006), 'On the value of mitigation and contingency strategies for managing supply chain disruption risks', *Management Science* **52**(5), 639–657.

Tomlin, B. T. (2009*a*), 'Disruption-management strategies for short life-cycle products', *Naval Research Logistics* **56**(4), 318–347.

Tomlin, B. T. (2009*b*), 'The impact of supply learning when suppliers are unreliable', *Manufacturing & Service Operations Management* **11**(2), 192–209.

Tomlin, B. T. and Wang, Y. (2005), 'On the value of mix flexibility and dual sourcing in unreliable newsvendor networks', *Manufacturing and Service Operations Management* **7**(1), 37–57.

Vakharia, A. J. and Yenipazarli, A. (2008), 'Managing supply chain disruptions', *Foundations and Trends in Technology, Information and Operations Management* **2**(4), 243–325.

Vanany, I., Zailan, S. and Pujawan, N. (2009), 'Supply chain risk management: Literature review and future research', *International Journal of Information Systems and Supply Chain Management* **2**(1), 16–33.

Wagner, S. M., Bode, C. and Koziol, P. (2009), 'Supplier default dependencies: Empirical evidence from the automotive industry', *European Journal of Operational Research* **199**(1), 150–161.

Wang, Y. and Tomlin, B. T. (2009), 'To wait or not to wait: Optimal ordering under lead time uncertainty and forecast updating', *Naval Research Logistics* **56**(8), 766–779.

Weiss, H. J. and Rosenthal, E. C. (1992), 'Optimal ordering policies when anticipating a disruption in supply or demand', *European Journal of Operational Research* **59**(3), 370–382.

Wilson, M. C. (2007), 'The impact of transportation disruptions on supply chain performance', *Transportation Research Part E* **43**, 295–320.

Wollmer, R. (1964), 'Removing arcs from a network', *Operations Research* **12**, 934–940.

Xia, Y., Yang, M.-H., Golany, B., Gilbert, S. M. and Yu, G. (2004), 'Real-time disruption management in a two-stage production and inventory system', *IIE Transactions* **36**(2), 111–125.

Xiao, T. and Yu, G. (2006), 'Supply chain disruption management and evolutionarily stable strategies of retailers in the quantity-setting duopoly situation with homogeneous goods', *European Journal of Operational Research* **173**, 648–668.

Xu, N. and Nozick, L. (2009), 'Modeling supplier selection and the use of option contracts for global supply chain design', *Computers and Operations Research* **36**(10), 2786–2800.

Xu, S. H., Tehrani, B. M., Kumara, S. and Li, H. (2011), 'A single period analysis of a two-echelon inventory system with dependent supply disruptions', *Transportation Research Part B* **45**, 1128–1151.

Yan, X. and Liu, K. (2009), 'An inventory system with two suppliers and default risk', *Operations Research Letters* **37**, 322–326.

Yang, Z., Aydin, G., Babich, V. and Beil, D. R. (2008), Supply disruptions, asymmetric information and a backup production option, Technical report, University of Michigan.

Yano, C. A. and Lee, H. A. (1995), 'Lot sizing with random yield: A review', *Operations Research* **43**(2), 311–334.

Yu, G. and Qi, X. (2004), *Disruption Management: Framework, Models, and Applications*, World Scientific Publishing Co., River Edge, NJ.

Yu, H., Zeng, A. and Zhao, L. (2009), 'Single or dual sourcing: decision-making in the presence of supply chain disruption risks', *Omega* **37**(4), 788–800.

Zhan, R. L., Daskin, M. S. and Shen, Z.-J. M. (2008), Facility reliability with site-specific failure probabilities. Working Paper, Department of Industrial Engineering and Operations Research, University of California-Berkeley.

Zipkin, P. H. (2000), *Foundations of Inventory Management*, Irwin/McGraw-Hill.