A reliable budget-constrained facility location/network design problem with unreliable facilities

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***Abstract*:** The combined facility location and network design problem is an important practical problem for locating public and private facilities. Moreover, incorporating aspects of reliability into the modeling of facility location problems is an effective way to hedge against disruptions in the system. In this paper, we consider a combined facility location/network design problem that considers system reliability. This problem has a number of applications, many of which fall into the category of service systems, such as regional planning and locating schools, health care service centers and airline networks. Our model also includes an investment budget constraint. We propose a mixed integer programming formulation to model this problem, as well as an efficient heuristic based on the problem's LP relaxation. Numerical results demonstrate that the proposed heuristic significantly outperforms CPLEX in terms of solution speed, while still maintaining excellent solution quality. Our results also suggest a favorable tradeoff between the "nominal cost" (including fixed facility location costs and link construction costs, as well as transportation costs) and system reliability; that is, substantial improvements in reliability are often possible with only slight increases in the total cost of investment and transportation.

Keywords: Facility location, Network design, Reliability, Investment budget constraint, Mixed integer programming, Heuristic.

1. Introduction

1-1. Motivation

Managers of service systems, as well as supply chains for goods, are continuously looking for ways to reduce total costs while also improving the performance of their systems in order to stay competitive in today's business landscape. At the same time, many companies, especially in industrialized countries, face disruptions and other unexpected events throughout their supply chains, and these can lead to disastrous financial losses. Combined with the worldwide economic downturn in recent years, this risk necessitates the use of proactive strategies for mitigating the effects of disruptions.

We present a model that combines facility location and network design decisions under the risk of disruptions. Thus, our model optimizes *strategic* decisions that account simultaneously for the need for efficiency (i.e., low costs) and for reliability (i.e., disruption resilience). (Other papers discuss *operational* approaches for mitigating disruptions; see, e.g., Tomlin (2006), or see Snyder et al. (2010) for a review.) Our model minimizes the total transportation cost assuming no disruptions take place while imposing a budget constraint on the fixed cost of building facilities and links in the network. It also ensures the reliability of the resulting network by enforcing an upper bound on the total cost that may result when disruptions occur.

Facility location problems choose the locations of facilities and, often, the allocation of customers to them, in order to optimize some objective function, such as minimizing the operating cost or maximizing the demands covered. Based on their objective functions and constraints, facility location problems are categorized into several problem classes, such as the *P*-median and *P*-center problems (Hakimi 1964), the uncapacitated facility location problem (Kuehn and Hamburger 1963), the maximum covering location problem (Church and ReVelle 1974) and the set covering location problem (Toregas et al. 1971). In network design problems, the basic goal is to optimally construct a network that enables some kind of flow, and possibly that satisfies some additional constraints. The nodes usually are given and the problem must make decisions about which links (edges) to choose from among a set of potential edges.

Often these two problems (facility location and network design) are solved independently, but we would argue that it is more realistic and effective to model and solve them simultaneously. All of the aforementioned classical facility location models locate facilities on a predetermined network. However, the topology of the underlying network may profoundly affect the optimal facility locations. Joint facility location/network design problems have many applications in industries and services, and some studies clearly illustrate the value of solving them simultaneously (Melkote 1996, Melkote and Daskin 2001).

Another significant subject that can affect facility location and network design is the reliability of the system. Assuming that facilities are always available and never disrupted is typical in classical studies. Although most companies would like to assume that disruptions rarely happen, and that even if they occur, their supply chains will be reliable enough, in practice, some unexpected disruptions happen and some companies are vulnerable and therefore easily disrupted. The terrorist attacks of 9/11, the catastrophic devastation caused by Hurricane Katrina (Barrionuevo and Deutsch 2005, Latour 2001, Mouawad 2005), the huge fines paid by the Boeing company in compensation for postponing the delivery of the Dreamliner 787 (Bathgate and Hayashi 2008, Peng et al. 2011) and the tragic earthquake and subsequent tsunami in Japan in 2011 (Bathgate and Hayashi 2008, Clark and Takahashi 2011) are among the most obvious examples of these kind of disruptions.

It can be difficult for companies to remove (or even reduce) the causes of disruptions, sometimes, because most of the times, the causes—such as equipment failures, natural disasters, industrial accidents, power outages, labor strikes, and terrorism—are out of companies' control and cannot effectively be avoided by precautionary actions. Although some disruptions are short-lived, they can still cause serious long-term negative financial and operational outcomes. Some studies have quantified these negative effects of disruptions empirically; for example, the abnormal stock returns of firms that have been affected by disruptions can reach approximately 40% (Hendricks and Singhal 2005). Similar findings are described by (Peng et al. 2011, Hicks 2002).

When facility disruptions occur, customers may have to be reassigned from their original facilities to the other available facilities, in which case the transportation costs will surely increase. Moreover, the facility locations that are chosen when the disruption risks are ignored may not be good locations to respond to disruptions; therefore, it is important to incorporate the risk of disruptions when making facility location and network design decisions. That is the primary focus of our study.

1-2. Literature Review

The initial model for the facility location/network design problem (FLNDP) was introduced by Daskin et al. in 1993 (Daskin et al. 1993). They demonstrated the importance of optimizing facility locations at the same time as network design and developed a mathematical model to do so. Subsequently, Melkote (1996) developed three models for the FLNDP in his doctoral thesis including uncapacitated and capacitated versions (UFLNDP and CFLNDP, respectively) and the maximum covering location-network design problem (MCLNDP). These models were also described by Melkote and Daskin (2001). In another doctoral thesis, some efficient approaches were developed to solve the static budget-constrained FLNDP by Cocking (2008). Also, Cocking (2008) developed some useful algorithms to find good upper lower bounds on the optimal solution. The main heuristics that were proposed in Cocking's doctoral thesis are simple greedy heuristics, a local search heuristic, metaheuristics including simulated annealing (SA) and variable neighborhood search (VNS), and a custom heuristic based on the problem-specific structure of FLND. In addition, a branch-and-cut algorithm using heuristic solutions as upper bounds, and cutting planes to improve the lower bound of the problem were developed. The method reduced the number of nodes which were needed to approach optimality.

Drezner and Wesolowsky (2003) proposed a new network design problem with potential links, each of which could be either constructed at a given cost or not. Also, each constructed link could be constructed as either a one-way or two-way link. Bigotte et al. (2010) studied a version of the FLNDP in which the multiple levels of urban centers and multiple levels of network links were considered simultaneously in developing of a mixed integer mathematical model. Their model determines the best transfers of urban centers and network links to a new level of hierarchy in order to improve the accessibility of all kinds of facilities. Jabalameli and Mortezaei (2011) proposed a bi-objective mixed integer programming formulation as an extension of the CFLNDP in which the capacity of each link for transferring the demands is limited. Contreras and Fernandez (2012) reviewed the relevant modeling aspects, alternative formulations and several algorithmic strategies for the FLNDP. They studied general network design problems in which design decisions to locate facilities and to select links on an underlying network are combined with operational allocation and routing decisions to satisfy demands. Contreras et al. (2012) presented a combined FLND problem to minimize the maximum customer-facility travel time. They developed and compared two mixed integer programming formulations by generalizing the classical P-center problem so that the models consider the location of facilities and the design of the underlying network simultaneously. Table 1 presents an overview of the literature on the FLND problem.

The literature related to system reliability in facility location problems demonstrates that, in light of the huge investment required for facility location, the attention paid to system failures in facility location has increased in recent years (Qi and Shen 2007, Qi et al. 2010). Drezner (1987) was one of the first researchers who proposed mathematical models for facility location with unreliable suppliers. He studied the unreliable P-median and (P,q)-center location problems, in which a facility has a given probability of becoming inactive. In subsequent research, Snyder and co-authors (2003, 2005, 2007) proposed several mathematical programming formulations for the reliable P-median and fixed charge problems based on level assignments, in which the candidate sites are subject to random disruptions with equal probability. Berman et al. (2007) formulated a P-median problem with disruptions that relaxes the equal-probability assumption made by Snyder and Daskin (2005). Their model is highly non-linear, and they focus on structural properties and special cases. Shen et al. (2009) also relaxed the assumption of uniform failure probabilities, formulated the stochastic fixed-charged facility location problem as a nonlinear mixed integer program, and proposed several heuristic solution algorithms, as well as a 2.674-approximation algorithm for the equal-probability case. Lim et al. (2009) proposed a reliability continuum approximation (CA) approach for facility location problems with uniform customer density in which facilities can be protected with additional investments. They demonstrated the impact of misestimating the disruption probability in facility location problems in the presence of random facility disruptions.

Hanley and Church (2011) developed a facility location–interdiction covering model for finding a robust arrangement of facilities that has a suitable efficiency under worst-case facility losses. They formulated a MIP model in which all possible interdiction patterns are considered, and a second, more compact bilevel model in which the optimal interdiction pattern is implicitly defined in terms of the chosen facility locations. Peng et al. (2011) studied the effect of considering of reliability in logistic networks design problems with facility disruptions and illustrated that applying a reliable network design is often possible with negligible increases in total location and allocation costs. They considered open/close decisions on nodes but not on arcs of the commodity production/delivery system. By applying the *p*-robustness criterion (which bounds the cost in disruption scenarios), they simultaneously minimize the nominal cost (the cost when no disruptions occur) and reduce the disruption risk. Recently, Liberatore et al. (2012) proposed a tri-level mathematical model for the problem of optimizing fortification plans in capacitated median distribution systems with limited protective resources in the face of disruptions that involve large regions. They illustrated empirically that considering correlation effects in a system plays an important role in reducing the suboptimal protection plans and subsequently decreasing the unessential growth in the system cost when disruptions happen. Moreover, Jabbarzadeh et al. (2012) studied a supply chain design problem in which distribution centers may have partial and complete disruptions. The problem was formulated as a mixed-integer nonlinear program which maximizes the total profit of the system while taking into account different disruption scenarios at facilities. Table 2 summarizes an overview on the literature of the facility location problems with respect to system reliability.

1-3.Model Overview

It is evident from the preceding literature review that the existing studies have not considered both network design and system reliability together with facility location. In fact, the literature review illustrates that there is a research gap in facility location regarding more realistic factors such as network design and system reliability to manage practical facility location problems. However, there are numerous examples of practical problems in which simultaneously considering facility location, network design, and system reliability is critical in improving the efficiency, usefulness, and security of the system. These examples include pipelines for gas and water, infrastructure for airline and railroad networks, and systems for delivering services such as health care and education. (In the latter example, link construction may represent establishing routes for medical transport vehicles or school buses, or may represent the construction of new roads to access the facilities, e.g., in underdeveloped regions.) Moreover, our model includes a budget constraint on the fixed cost of locating nodes and links in the network, which reflects a practical constraint faced by many of these systems.

We will refer to our model as the reliable budget-constrained facility location/network design problem (RBFLNDP). The main contributions that differentiate this paper from the existing ones in the related literature can be summarized as follows: (1) We introduce a new optimization model to consider simultaneously facility location and allocation, network design, system reliability and a budget constraint as a mixed-integer, linear programming (MILP) problem. Our model integrates tactical and strategic decision making, such as determining the optimum locations of new facilities, optimum construction of transportation links, and optimum allocation of demand nodes to located facilities so that total costs as well as system reliability are optimized. (2) Our new mathematical formulation not only takes into account facility location costs, link construction costs, and transportation costs, but also constrains the maximum allowable disruption cost of the system, as well as the investment in facility location and transportation link construction. (3) We develop a new hybrid heuristic solution approach for the RBFLNDP that, to the best of our knowledge, has not previously been proposed for solving facility location problems.

The remainder of the paper is organized as follows: In Section 2, the mathematical formulation of the RBFLNDP is developed. In Section 3, the hybrid LP relaxation heuristic solution approach is proposed and described. Then, in Section 4, a numerical example that illustrates the application of the heuristic is demonstrated and, based on it, a sensitivity analysis of the model parameters is reported. Computational results are presented in Section 5 and finally, conclusions and future works are discussed in Section 6.

Table 1: An overview on the literature of the FLND problem

Authors	Year	Contribution(s)	Assumptions and descriptions	Solution method(s)	Other comments
Daskin et al.	1993	• Considering the facility location & network design topics simultaneously for the first time • Proposing mathematical model for facility location/network design problem (FLNDP)	• Locating uncapacitated facilities and optimizing network design simultaneously	-	• Proposing a mixed integer mathematical model for FLNDP
Melkote and Daskin	1996, 2001	• Proposing several mathematical models for FLNDP with different assumptions	• Minimizing the facility location, link construction and transportation costs for: ✓ Uncapacitated facilities ✓ Capacitated facilities • Maximizing the covering of demand nodes by located facilities in FLND problem	• Branch and bound approach • Heuristic algorithm	• Proposing three mathematical models
Drezner and Wesolowsky	2003	• Considering FLNDP as two separate objective functions (facility location costs & traffic flow costs)	• Designing and constructing the links as one direction & two directions with different costs	• Heuristic algorithms • Simulated annealing • Tabu search • Genetic algorithm	• Simultaneously optimizing facility location and network design as two separate objective functions
Miranda	2004	• Modeling of FLNDP with congestion costs and interdependency	• Considering congestion costs • Bringing up the interdependency among facilities	• Benders decomposition algorithm • Branch and bound approach in CPLEX solver	-
Cocking	2008, 2009	• Modeling of budget-constrained FLNDP	• Considering of investment budget constraint for facility location & network design	• Branch and cut algorithm • Heuristic algorithm • Variable neighborhood search (VNS) including RVNS & VNDS • Simulated annealing	• Applying the proposed solution methods for a real case study
Bigotte et al.	2010	• Integrated modeling of urban hierarchy and transportation network planning	• Considering facility location and allocation as hierarchical and multi-level problem	• Heuristic algorithm • Nested partitioning algorithm • Tabu search • Genetic algorithm	• Applying the proposed solution methods for a real-world application of urban hierarchy and transportation network planning
Jabalameli and Mortezaei	2011	• An extension of the FLNDP proposed by Melkote (1996, 2001) subject to capacitated facilities as multi objective mathematical model	• The facilities are capacitated • The transportation links are capacitated	• Hybrid heuristic algorithm • Lexicography method	-
Contreras and Fernandez	2012	• Discussing and analyzing general network design problems subject to combining design decisions to locate facilities and to select links on an underlying network with the operational allocation and routing decisions to satisfy demands	• Considering the relevant modeling aspects, alternative formulations and possible algorithmic strategies of general network design problems	-	-
Contreras et al.	2012	• Minimizing the maximum customer-facility travel time of center FLNDP	• Generalizing the classical *P*-center problem for various applications in regional planning, distribution • Formulating multi-commodity-type decision variables for telecommunications, emergency systems	• Branch and cut algorithm • Heuristic algorithm	-

Table 2: An overview of the literature on facility location problems considering system reliability

Author(s)	Year	Contribution(s)	Assumptions and descriptions	Solution method(s)	Other comments
Drezner	1987	• Considering disruptions in a facility location problem for the first time	• Introducing two mathematical models: ✓ First, a reliability version of the classical *P*-median problem, assuming that nodes fail with a given probability. The failure probability for each node is known, as are the joint probabilities. The objective is to minimize the expected demand-weighted travel distance. ✓ Second, the (*P*; *q*)-center problem, a type of *P*-center problem in which *P* facilities must be located to minimize the maximum cost that may occur when at most *q* facilities fail.	• Variable neighborhood search • Heuristic algorithm	• Providing small illustrative examples demonstrating the behavior of the (*P*; *q*)-center problem
Snyder	2003, 2005	• Proposing mathematical models for PMP & UFLP considering the failures of facilities	• Formulating models for choosing facility locations to minimize cost while also taking into account the expected transportation cost after failures of facilities. • The goal is to choose facility locations that are both inexpensive under traditional objective functions and also reliable.	• Exact Lagrangian relaxation algorithm	-
Berman et al.	2007	• Study of facility reliability issues in network *P*-median problems subject to strategic centralization and co-location effects	• Analyzing a facility location model where facilities may be subject to disruptions depending on the probability of facility failure. • Generalizing the classical *P*-median problem on a network to explicitly include the failure probabilities.	• Greedy heuristic method	• Illustrative case study: location hospitals in Toronto, Canada
Shen, et al.	2007	• Study of the reliable uncapacitated facility location problem in which the failure probabilities are site-specific. • Developing several models that can be used to fortify the reliability of the existing facilities.	• Formulating the problem as a two-stage stochastic program and then a nonlinear integer program.	• Monotonic branch-reduce-bound algorithm, • Sample average approximation heuristic • Four heuristic algorithms • Genetic algorithm based heuristic algorithm • Approximation algorithm with worst case bound for the special case where the failure probability at each facility is the same	-
Lim et al.	2009	• Proposing a reliable facility location design in the presence of random facility disruptions with the option of hardening selected facilities	• Formulating a facility location problem as a mixed integer programming model incorporating two types of facilities, one that is unreliable and another that is reliable	• Lagrangian relaxation-based solution algorithm	-
Gade and Pohl	2009	• Developing a mathematical model for discrete capacitated-facility location problem	• Presenting a stochastic programming formulation that deals with opening facilities with a finite capacity to serve a set of customers.	• Sample average approximation-based algorithm	
Cui et al.	2010	• Proposing a compact mixed integer program formulation and a continuum approximation (CA) model to study the reliable uncapacitated fixed charge location problem	• Determining the optimal facility locations as well as the optimal customer assignments to minimize initial setup costs and expected transportation costs in normal and failure scenarios.	• Lagrangian relaxation algorithm • Continuum approximation (CA) method for large scale problems	
Hanley and Church	2011	• Developing a facility location–interdiction model in order to design a coverage-type service network that is robust to the worst instances of long-term facility loss	• Formulating the problem both as a mixed-integer program and as a bilevel mixed-integer program that maximizes a combination of initial coverage by *P* facilities and the minimum coverage level following the loss of the most critical facility	• Bilevel programming algorithm • Bilevel decomposition algorithm	
Peng et al.	2011	• Study of reliable logistics networks design with facility disruptions	• Proposing a mixed-integer programming model to simultaneously minimize the nominal cost and reduce the disruption risk using the *p*-robustness criterion	• Genetic algorithm-based heuristic	• Evaluating the efficiency of hybrid metaheuristic algorithm vs. branch and bound algorithm
Liberatore et al.	2012	• Considering the problem of optimally protecting a capacitated median system with a limited amount of protective resources subject to disruptions.	• The type of disruption studied is characterized by correlation effects between the facilities, and may result in partial or complete disruption of the facilities involved. • The model optimizes protection plans in the face of large area disruptions.	• Exact solution algorithm which makes use of a tree-search procedure to identify which facilities to protect	• Testing the algorithm on a dataset based on the 2009 L'Aquila earthquake.
Jabbarzadeh et al.	2012	• Study of a supply chain design problem considering the risk of disruptions at facilities	• Study of supply chain design in which distribution centers may have partial and complete disruptions. • Formulating the problem as a mixed-integer nonlinear program which maximizes the total profit for the whole system while taking into account different disruptions scenarios at facilities.	• Lagrangian relaxation method • Genetic algorithm	

2. Mathematical Formulation of the RBFLNDP

2-1. Problem Description

We assume that we are given a set of demand nodes, as well as a set of potential transportation links among them. Each of the demand nodes is also a candidate facility node, and our goal is to choose facility nodes and transportation links, thereby constructing a transportation network to meet all of the demand. Fixed costs are incurred for constructing nodes and links, and transportation costs are incurred for each unit of demand that flows along the links. If some demand nodes are not eligible to become facilities, we can set their fixed costs to infinity, and if some facilities or links already exist, we can set their fixed costs to zero. The total investment cost (for locating facilities and constructing links) has a predetermined upper bound, represented in our model by a budget constraint. An additional upper bound is imposed on the investment by requiring that no more than P facilities are constructed. (By including both fixed costs and a limit of P facilities, our model combines aspects of both the uncapacitated fixed cost location problem and the P-median problem.)

Moreover, we assume that the facilities are not reliable and, due to unexpected events such as poor weather or sabotage, they occasionally fail and become unavailable. Accordingly, the demand nodes that were served by the disrupted facility must be reassigned to the nearest active facility. Of course, the re-assigned flows to the backup facilities are not optimal, leading to increased transportation costs, as well as increased link construction costs to accommodate the rerouted flows. The cost that is incurred during a disruption is known as the "failure cost" (JabalAmeli and Mortezaei 2011, Contreras and Fernandez 2012), and its upper bound may be called the "maximum allowable failure cost." Our goal is to bound the failure cost that occurs for any disruption, regardless of how likely the disruption is. Accordingly, we do not consider either the probability or the duration of disruptions.

We assume that the system functions as a *customer-to-server* system in which customers themselves travel to the facilities in order to receive service. Thus, when we speak of a "flow" on a given link, we are speaking of the flow of *customers* traveling on the link toward the facility they patronize. In contrast, many facility location or network design models treat flows of *goods*, and in the opposite direction, from facilities toward customers. The *customer-to-server* assumption is common for service systems and is consistent with the models proposed by Melkote (1996). However, this assumption is not critical for our model, which could be easily adapted to accommodate flows in the opposite direction through appropriate modifications to the parameters, decision variables, and constraints.

In our model, when we choose the location of a facility, we also choose which facility will serve as its backup when the facility is disrupted. Note that this differs from the notion of "backup facility" as used by Snyder and Daskin (2005) and other authors, in which backups are assigned at the customer level, not the facility level. That is, in other models, two customers assigned to the same facility may have different backup facilities, whereas in our model, they have the same backup facility.

Suppose that node k is chosen as the backup facility for a facility at node i. We assume that additional links must be constructed to accommodate the new flows into node k when a disruption occurs at node i. Thus, to accommodate rerouting when i is disrupted, a link (j,k) must be constructed for every link (j,i) that was constructed for normal flows, and a link from i to k must also be constructed (for the demands originating at i).

To summarize, our problem is to determine: (1) the optimum locations of facilities; (2) the primary facility and backup facility of every demand node; (3) the transportation links that should be constructed for both normal and disrupted conditions; and (4) the amount of demand that should be transported on each transportation link. The objective function minimizes the transportation cost, while constraints bound the investment cost and the failure cost.

2-2. Additional Assumptions

In addition to the assumptions described in the preceding section, we assume the following:

- ✓ The facilities and network links are uncapacitated.
- ✓ Facilities can only be located on the nodes of the network, not on links.
- ✓ All travel costs are symmetric.
- ✓ All network links are reliable; that is, disruptions occur at the nodes only.
- ✓ At most one facility fails at a time.

2-3. Notation

Sets and Parameters:

N	set of nodes in the network; each is both a demand node and a potential facility location
S	set of candidate links in the network
d_i	demand at node $i \in N$
D	$\sum_{i \in N} d_i$ = total demand
B	investment budget for (i.e., upper bound on) facility location and link construction
f_i	fixed cost of locating a facility at node $i \in N$

c_{ij} cost of constructing link (i, j)
P number of facilities to open, $(P \geq 2)$
FC maximum allowable failure cost
t_{ij}^{0} transportation cost of a unit of flow on link (i,j)
t_{ij}^{l} transportation cost for all of the demand of node l to flow on link $(i,j) = t_{ij}^{0}\, d_l$
M large number

Note that t_{ij}^{0} and t_{ij}^{l} represent link-specific transportation costs, not origin-destination transportation costs. Since customers traverse routes that consist of multiple links (unlike in classical location problems), we must model the flows link-by-link and therefore use link-specific transportation costs.

We assume that the maximum allowable failure cost, FC, is the same for all facilities, for the sake of simplicity. The model can easily be modified to allow facility-dependent maximum allowable failure costs by appending a subscript to FC and modifying the appropriate constraint in the formulation below. Determining a suitable value for FC in practice may be challenging, because firms and service organizations may find it difficult to quantify the specific maximum failure cost they could withstand. However, our problem can be solved iteratively with different values of FC to obtain a tradeoff curve from which decision makers may choose a solution that strikes an appropriate balance between operating cost and failure cost, based on their preference. The method for generating this tradeoff curve is discussed in Section 4.

Decision variables:

$Z_{ik} = 1$ if a facility is located at node i and the facility located at node k is node i's backup facility, 0 otherwise
$X_{ij} = 1$ if link (i,j) is constructed, 0 otherwise
Y_{ij}^{l} = fraction of demand of node l that flows on link $(i,j) \in S$
$Y_{ij}^{i} = X_{ij} \quad (i,j)\in S$
W_i^{l} = fraction of demand of node l that is served by a facility at node $i\in N$
$W_i^{i} = \sum_{k\in N} Z_{ik} \quad i\in N$

Since a backup facility is required for each open facility, a facility is located at node i if and only if $\Sigma_k Z_{ik} = 1$. (It is allowable for Z_{ik} and Z_{ki} both to equal 1. Each facility may serve as both a "primary" facility and as a backup for another facility.)

The transportation (flow) variables work as follows. Y_{ij}^{l} represents the fraction of node l's demand that flows on link (i,j). (Recall that "flow" refers to the flow of customers toward their facilities.) No flow is allowed on (i,j) unless that link is constructed, i.e., unless $X_{ij} = 1$. Moreover, if link (i,j) is constructed, then we assume that the demand of node i flows on it. (This is optimal if the link construction and transportation costs satisfy the triangle inequality.) Thus, we define $Y_{ij}^{i} = X_{ij}$. A second set of flow variables, W_i^{l}, indicates which facilities (i) serve which customers (l), ignoring which links that flow actually travels on. We assume that if i is selected as a facility, then that facility serves the demand at i itself; hence, $W_i^{i} = \Sigma_k Z_{ik}$.

2-4. Formulation

With respect to the above assumptions and notations, the mathematical formulation of the RBFLNDP is shown below:

(RBFLNDP)
$$Min\ C = \sum_{(i,j)\in S} t_{ij}^{i} X_{ij} + \sum_{(i,j)\in S} \sum_{l\in N: l\neq i} t_{ij}^{l} Y_{ij}^{l} \tag{1}$$

$s.t.$

$$\sum_{i\in N}\sum_{k\in N} f_i Z_{ik} + \sum_{(i,j)\in S} c_{ij} X_{ij} \leq B \tag{2}$$

$$\begin{aligned}&\left[\sum_{m\in N}\sum_{n\in N} f_m Z_{mn} + \sum_{(m,j)\in S} c_{mj} X_{mj} + \sum_{(m,j)\in S} t_{mj}^{m} X_{mj} + \sum_{(m,j)\in S}\ \sum_{l\in N: l\neq m} t_{mj}^{l} Y_{mj}^{l}\right] \\ &+\left[c_{ik} + \sum_{j\in N} c_{jk} X_{ji} + \sum_{j\in N} (t_{jk}^{i} - t_{ji}^{i}) X_{ji} + \sum_{j\in N}\ \sum_{l\in N: l\neq i} (t_{jk}^{l} - t_{ji}^{l}) Y_{ji}^{l}\right] \leq FC + M(1 - Z_{ik}) \quad \forall i, k\in N\end{aligned} \tag{3}$$

$$X_{li} + \sum_{j\in N: j\neq l} Y_{ji}^{l} = \sum_{j\in N} Y_{ij}^{l} + W_i^{l} \qquad \forall i, l \in N: i\neq l,\ (l, i)\in S \tag{4}$$

$$\sum_{j\in N: j\neq l} Y_{ji}^{l} = \sum_{j\in N} Y_{ij}^{l} + W_i^{l} \qquad \forall i,l\in N: i\neq l,\ (l,i)\notin S \tag{5}$$

$$\sum_{k\in N} Z_{ik} + \sum_{j\in N} X_{ij} = 1 \qquad \forall i\in N \tag{6}$$

$$\sum_{k\in N} Z_{ik} + \sum_{j\in N: j\neq i} W_j^{i} = 1 \qquad \forall i\in N \tag{7}$$

$$Y_{ij}^{l} \le X_{ij} \qquad \forall (i,j)\in S,\ \forall l\in N: i\neq l, \tag{8}$$

$$W_i^{l} \le \sum_{k\in N} Z_{ik} \qquad \forall i,l\in N: i\neq l \tag{9}$$

$$\sum_{i\in N}\sum_{k\in N} Z_{ik} = P \tag{10}$$

$$\sum_{k\in N} Z_{ik} \le 1 \qquad \forall i\in N \tag{11}$$

$$\sum_{i\in N} Z_{ik} \le \sum_{m\in N} Z_{km} \qquad \forall k\in N \tag{12}$$

$$Z_{ii} = 0 \qquad \forall i\in N \tag{13}$$

$$Y_{ij}^{l} \ge 0 \qquad \forall i,j,l\in N \tag{14}$$

$$W_i^{l} \ge 0 \qquad \forall i,l\in N \tag{15}$$

$$X_{ij}\in\{0,1\} \qquad \forall (i,j)\in S,\ \forall l\in N: l\neq i \tag{16}$$

$$Z_{ik}\in\{0,1\} \qquad \forall i,k\in N: l\neq i \tag{17}$$

The objective function (1) includes the transportation costs on all transportation links. The first term represents the first "leg" of the flow from demand node *i*, while the second term represents any additional links that node *i*'s demand travels along. Constraint (2) stipulates that the investment in facility location and link construction cannot exceed *B*. Constraints (3) are the reliability constraints. If a facility is opened at *i* and has backup facility *k* (i.e., $Z_{ik} = 1$), then the total failure cost may not be greater than *FC*. The first bracket calculates the "nominal" cost (the location and transportation cost if no disruptions occur), while the second bracket calculates the increase in cost when facility *i* fails. In particular, when facility *i* fails we must construct a new link to *k* from each node *j* for which a link (*j*,*i*) exists for normal conditions, as well as a new link from *i* to *k* to accommodate *i*'s demand; these costs are represented by the first two terms in the second bracket. We must also re-route the flows on these new links, and the *additional* transportation cost from this re-routing is represented by the second two terms inside the second bracket. Note that if $Z_{ik} = 0$, then the constraint is non-binding since the right-hand side is large.

Constraints (4) and (5) are flow-conservation constraints requiring that, for each pair of nodes *l* and *i*, the flow of node *l*'s demand into node *i* equals the flow of the same out of node *i* plus the demand served by a facility at node *i*, if any. Note that the customer-to-server assumption means that we treat facility nodes as "sinks" for the demand. The two constraints differ by the term X_{li}, which is included in the inbound flow if (*l*,*i*) is a potential link (in which case it equals Y_{li}^{l}).

Constraints (6) ensure that, for each node *i*, either there is a facility at *i* or some link is constructed out of *i*. Constraints (7) require the demand of node *i* to find a destination, whether it is satisfied by node *i* itself ($Z_{ik} = 1$ for some *k*) or by some other node *j* ($W_j^{i} = 1$). Constraints (8) and (9) guarantee that potential links and facilities are not used if they are not constructed. Constraint (10) restricts the total number of newly located facilities to *P*. Constraints (11) stipulate that at most one facility *k* may be chosen as the backup for *i*. Constraints (12) prevent facility *k* from being used as a backup for facility *i* if *k* has not been opened. Constraints (13) say that a facility cannot be selected its own backup facility. Constraints (14)–(17) are standard non-negativity and integrality constraints. Note that, although we define *Y* and *W* as continuous variables, there exist optimal solutions in which they are binary, as in many other uncapacitated facility location models. Thus, we can treat the demand of each node as though it is completely assigned to the closest single facility rather than split among multiple facilities.

2-5. Complexity

Property 1 establishes that the RBFLND problem is NP-hard, since it has the *P*-median problem, which is itself NP-hard, as a special case.

Property 1: *The RBFLND problem is NP-hard.*

Proof: The RBFLNDP reduces to the classical P-median problem if we set $f_{ij} = c_{ij} = 0$ and $B = FC = \infty$. Since the P-median problem is NP-hard (Kariv and Hakimi 1979), so is the RBFLNDP. (Note that the RBFLNDP allows travel through intermediate nodes, whereas the classical P-median assumes direct travel between customers and facilities. However, the model in Kariv and Hakimi (1979) assumes distances are calculated as shortest paths in an underlying network, and therefore the two interpretations are identical.) □

3. Solution procedure: LP relaxation heuristic approach

We coded the formulation for the RBFLNDP proposed in Section 2-5 in GAMS 23.3 and solved it using CPLEX 12. We found that CPLEX can find an optimal solution for the RBFLNDP quickly for small-scale instances but that the run times increase quickly as the problem size grows, as suggested by Property 1. Thus, it is desirable to have an efficient heuristic solution procedure to solve larger-scale instances of the RBFLNDP. From the literature review in Section 1-2, one can conclude that customized heuristics based on problem-specific structure have played an important role in solving both the FLND problem and reliable facility location problems, and that they can often obtain near-optimal solutions to these problems in a reasonable computation time.

Motivated by this, we propose a new LP relaxation-based heuristic to solve the RBFLNDP. The basic idea is to first solve the LP relaxation of (RBFLNDP), then to round the resulting location variable matrix (Z^{LP}) to integers, and finally to solve the original (RBFLNDP) model but with the location variables fixed to these new integer values. The rounding is performed based not on the individual elements Z_{ij}^{LP} of Z^{LP}, but on the sums $Z_{ij}^{LP} + Z_{ji}^{LP}$, since this sum provides more information about whether i and j represent a good *pair* of facilities. The heuristic proceeds as follows:

1. Solve the LP relaxation of (RBFLNDP). Let Z^{LP} denote the $n{\times}n$ matrix of location variables, and let Z_{ij}^{LP} be its (i,j)th element.
2. Improve Z^{LP} heuristically to obtain a binary matrix Z^{imp} as follows:
    a. Let Z^{LP2} be a new upper-triangular matrix in which $Z_{ij}^{LP2} = Z_{ij}^{LP} + Z_{ji}^{LP}$ if $i < j$ and $Z_{ij}^{LP2} = 0$ if $i > j$. (Recall that $Z_{ij}^{LP2} = 0$ if $i = j$.)
    b. Let Z^{imp} be an $n{\times}n$ matrix consisting of all 0s , $\Psi = \{\emptyset\}$ and $\theta = 0$ (Note that θ represents the number of elements of Ψ).
    c. While $\theta \leq P$ do:
        i. Let (i,j) be the largest element of Z^{LP2}. If there are multiple elements with the same maximum value, select the one whose column has the largest sum.
        ii. Set the values of Z_{ij}^{LP2} and Z_{ij}^{imp} based on the "*If-Then*" conditions described in Fig. 1.

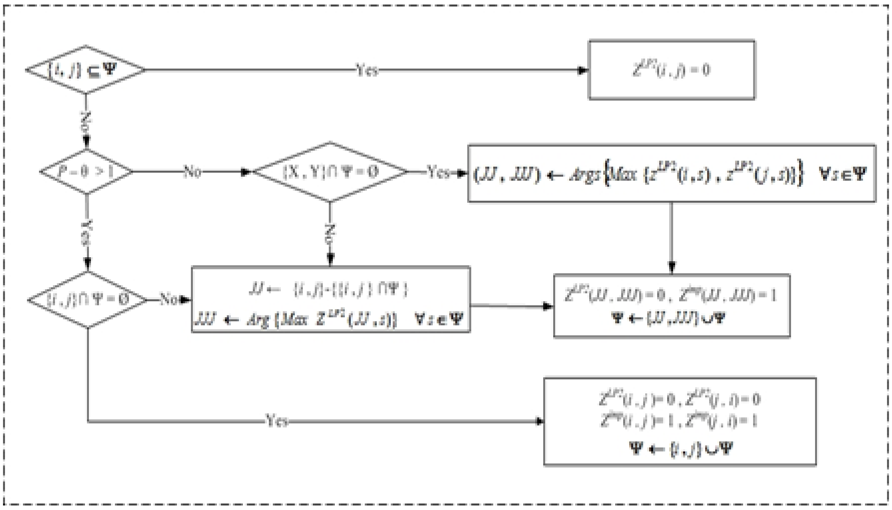

Fig. 1: "*if-then*" conditions flowchart of heuristic algorithm

3. Solve (RBFLNDP), treating the decision variables Z as fixed parameters equal to the corresponding values in Z^{imp}.

We consider a small numerical example to illustrate the heuristic. Suppose that Z^{LP}, the output of the LP relaxation of (RBFLNDP), is as given below. By adding the corresponding values as in step 2(a), we obtain the matrix Z^{LP2}.

$$Z^{LP} = \begin{bmatrix} 0 & 0.341 & 0.318 & 0.341 & 0.318 & 0.341 \\ 0.341 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.318 & 0 & 0 & 0 & 0 \\ 0.341 & 0.113 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.341 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow Z^{LP2} = \begin{bmatrix} 0 & 0.682 & 0.318 & 0.682 & 0.318 & 0.341 \\ 0 & 0 & 0.318 & 0.113 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.341 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Final\ Z^{LP2} = \begin{bmatrix} 0 & 0 & 0.318 & 0 & 0 & 0.341 \\ 0 & 0 & 0.318 & 0.113 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.341 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The maximum value of Z^{LP2} is 0.682, but elements (1,2) and (1,4) both attain this maximum. Since column 2 sums to 0.682 and column 4 sums to 0.795, we select element (1,4), and we set $Z_{1,4}{}^{imp} = 1$ and $Z_{1,4}{}^{LP2} = 0$. We repeat this process until Ψ contains P elements. If $P = 4$, the final matrix Z^{imp} has elements (1,4), (4,1), (2,1), and (5,1) equal to 1. In other words, facilities are opened at nodes 1, 2, 4, and 5, and the variables Z_{14}, Z_{41}, Z_{21}, and Z_{51} are fixed to 1 when we solve (RBFLNDP).

4. Numerical Example

We now provide a numerical example illustrating the application of the model and heuristic to a 21-node example used by several authors (Melkote 1996, Daskin 1987, Hodgson M J, Rosing 1992, Simchi-Levi D, Berman 1988). The network has 38 potential links that may be selected for construction. Other data, such as the facility location cost, the demand of each node and the transportation cost of each link are described in Melkote (1996). Also, a fixed coefficient u is defined as the cost of constructing one unit length of a link, so that each link construction cost is calculated as $c_{ij}=u\ t_{ij}$. We assume that $P = 4$, $u = 30$ and the budget B = \$25,000.

We begin by ignoring the disruptions (setting $FC = \infty$). The optimal solution for this problem, obtained by solving (RBFLNDP) optimally using CPLEX, is depicted in Fig. 2. The optimal facilities are 2, 10, 12, and 18, and the optimal links are displayed in the figure. Note that some demands (e.g., nodes 4 and 11) are served directly by a facility, while others (e.g., nodes 5 and 13) are served via intermediate nodes and links.

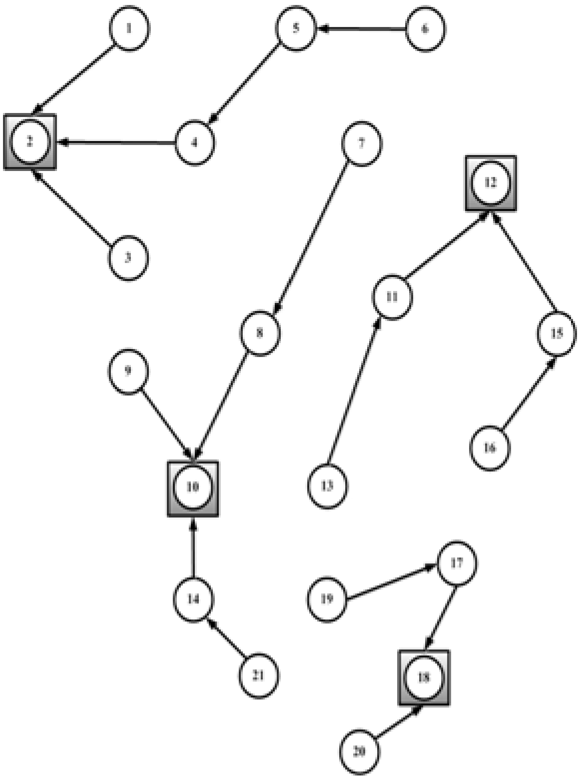

Fig. 2: The optimal solution of the numerical example without considering disruptions (*FC*=∞)

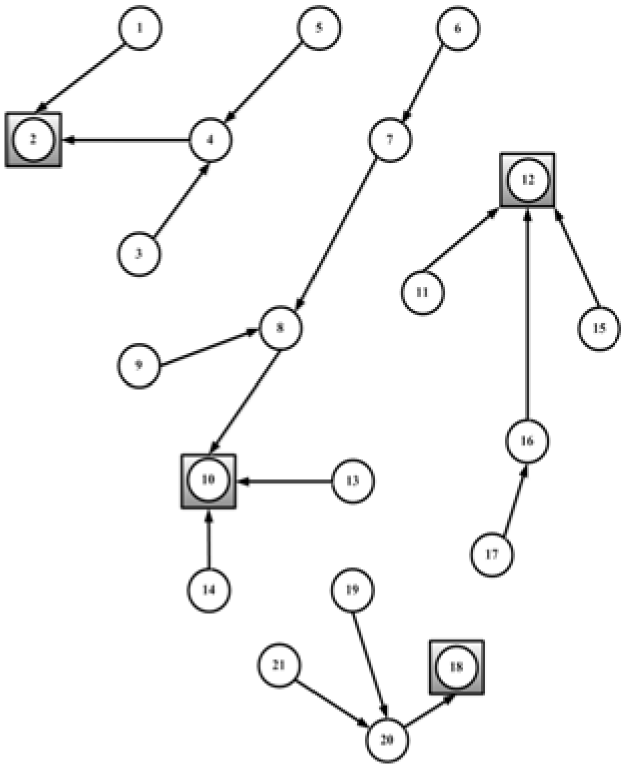

Fig. 3: The optimal solution of the numerical example considering disruptions (*FC*=\$159,000)

The total cost of this solution, which ignores disruptions, is $28,410. However, suppose that the worst-case disruption occurs; this happens to be facility 18, which has the maximum failure cost (i.e., the largest left-hand side of constraints (3)). In this case, the demand nodes that are served by facility 18 must be served by facility 18's best (or nearest) backup facility, facility 10. This results in a failure cost of $168,666.00. (Recall that the objective function includes transportation costs only, whereas the failure cost includes all costs.)

Now suppose, instead, that the decision makers want the maximum value of the failure cost to be not more than $159,000 ($FC$ = $159,000). In this case, the optimal design, shown in Fig. 3, has a cost of $32,310 when there is no disruption. But, in this case, if there is a disruption at the facility located at node 10 (which has the maximum failure cost), the demand nodes that are served by node 10 are fulfilled by facility 10's backup facility, facility 2 (according to the obtained solution of RBFLNDP model), and the failure cost is $158,256. (Note that node 2 may not be the best (or nearest) backup facility for node 10. However, it is good enough to ensure that the failure cost is less than the upper bound of $159,000.000. In general the model does not differentiate among these "good enough" backup facilities, and any of them may be chosen.) Clearly, the new solution causes a small increase in transportation cost versus the original solution ($32,310 versus $28,410), but the maximum failure cost is significantly reduced ($158,000 versus $168,666). This reduction can be critical, especially in emergency conditions.

To provide insights into the behavior of the objective function of (RBFLNDP) in response to changes in FC and B, Fig. 4 and Fig. 5 present tradeoff curves for the RBFLNDP for the problem instance from Melkote (1996). Fig. 4 shows how the objective function changes with FC. The optimal FLNDP solution ($FC = \infty$) is the left-most point on the curve, and subsequent points represent solutions obtained by choosing other values of FC. Evidently, the objective function decreases as the maximum failure cost increases; there is a tradeoff between the two. This relationship is logical because, in order to increase the reliability of the network, additional facility location costs, link construction costs, and transportation costs must be paid. Fortunately, the left part of the tradeoff curve is steep, indicating that large improvements in reliability may be attained with small increases in FLNDP cost. For example, the fourth point on the curve has a 6% larger value of C^* versus the optimal solution to the FLNDP but a 20% reduction in the maximum failure cost. The smooth right-most portion of the curve is of less interest, because it shows a large increase in the total cost compared with a very small decrease in the maximum failure cost.

Another important factor affecting the value of objective function is the investment budget B. Fig. 5 illustrates the changes in the optimal value of the objective function (C^*) for different values of B. From Fig. 5, it is clear that the optimal value of C^* will increase considerably as the value of B decreases. This relationship is also logical, because as the investment budget decreases, fewer facilities and links can be constructed, and therefore travel costs will increase.

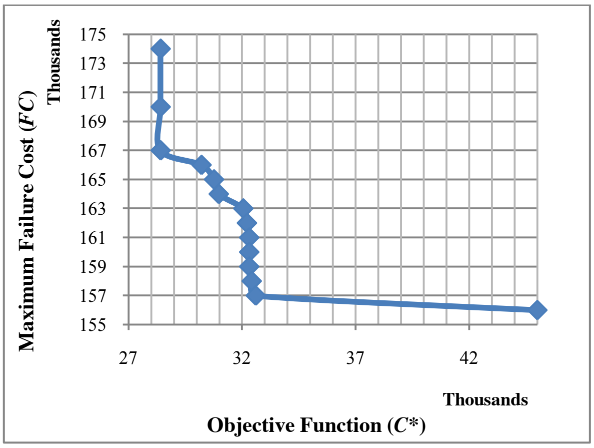

Fig. 4: The changes in optimal value of objective function for different values of FC

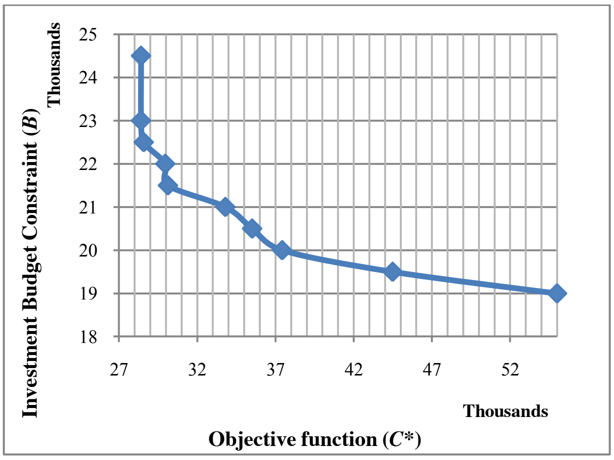

Fig. 5: The changes in optimal value of C^* for different values of B

5- Computational Results

A series of numerical experiments to evaluate the performance of the proposed heuristic approach were performed. The algorithm was coded in MATLAB R2011b and GAMS 23.3.3 and executed on a computer with an AMD Opteron 2.0 GHz (×16) processor and 32GB RAM, operating under Linux.

5-1. Experimental Design

In order to verify the performance of the proposed heuristic approach, we solved 30 problems of varying sizes. These problems were generated randomly in a manner similar to that described in the literature (Melkote

1996, 2001). In particular: The transportation cost for each link was randomly drawn from a discrete uniform distribution on [30, 100]. The construction costs of new links were calculated by multiplying the transportation cost by a coefficient *u*, where *u* has a discrete uniform distribution on [15, 30] and is drawn once for each instance and used for all links. The demand at each node and the fixed cost of opening each facility were sampled uniformly from [10, 150] and [1200, 3000], respectively, and then rounded to the nearest integer. Our instances contain between $N = 5$ and 60 nodes and P varies from 2 to 9.

We set CPLEX's optimality tolerance to 10% and its time limit to 2500 seconds for both the exact and heuristic methods. That is, CPLEX terminated when either of these criteria were reached, both when solving the problem optimally and when solving the IP in the final step of the heuristic.

5-2. Algorithm Performance

Table 3 summarizes the performance of the proposed heuristic algorithm with that of CPLEX. For each algorithm, the table represents the run time ("Time") and objective value ("Cost"). The run time for the heuristic includes the time required to solve the LP relaxation in step 1, which is then used as an input for the main step of the algorithm. The table reports the lower bound from CPLEX ("CLB") and from the heuristic ("HLB"), where the latter represents the objective value of the LP relaxation solved in step 1 of the heuristic, and the percentage gap between the objective value of the best solution found and the corresponding lower bound:

$$Gap_{CPLEX}(\%) = \frac{Cost_{CPLEX} - CLB}{Cost_{CPLEX}} \times 100 \quad ; \quad Gap_{Heuristic}(\%) = \frac{Cost_{Heuristic} - HLB}{Cost_{Heuristic}} \times 100$$

Finally, the last two columns give the ratio between the computation times (solution costs, respectively) of the two methods:

$$Ratio_{Time}(\%) = \frac{Time_{Heuristic}}{Time_{CPLEX}} \times 100 \qquad ; \qquad Ratio_{Cost}(\%) = \frac{Cost_{Heuristic}}{Cost_{CPLEX}} \times 100$$

Values less than 100% in the "Time (%)" and "Cost (%)" columns indicate that our heuristic outperformed CPLEX with respect to CPU time and solution cost, respectively. (CPLEX may find sub-optimal solutions because of our termination settings, as described above.) Our heuristic was faster than CPLEX for all instances. In addition, the notation "n/a" in the table indicates that no feasible solution could be obtained for that instance in the allowed time (2500 seconds).

The proposed heuristic algorithm was able to find the same or better solutions than CPLEX for 25 of the 36 test problems (69.4%). The heuristic is also much faster: it required only 64.6% of CPLEX's time, on average, and found solutions within the time limit for all but one of the instances, whereas CPLEX failed to do so for 9 of the 36.

Fig. 6 illustrates the run times of the proposed heuristic algorithm and CPLEX graphically. Each data point represents the average of the three instances (each with a different value of *P*) for each value of *N*. From Fig. 6, it can be concluded that the CPU time of CPLEX increases sharply as the number of nodes increases; moreover, CPLEX cannot solve the instances with more than 50 nodes to 10% optimality. In contrast, the proposed heuristic can obtain the same or better solutions in a reasonable time compared with CPLEX.

6. Conclusions and future research

In this paper, we considered the combined facility location/network design problem considering two additional aspects not previously included in studies of this problem, namely, system reliability and budget constraints. Our problem, called the reliable budget-constrained facility location/network design problem (RBFLNDP), was formulated as a mixed integer linear programming model. The basic principal in the proposed formulation is the concept of "backup" assignments, which indicate the backup facilities to which clients are assigned when closer facilities have failed and are not available. The tradeoff between the nominal cost and system reliability emphasized that significant improvements in system reliability are often possible with slight increases in the total cost. Moreover, a sensitivity analysis was done to provide insight into the behavior of the proposed model in response to changes in the reliability limit and the investment budget. The sensitivity analysis for the maximum allowable failure cost indicates that large improvements in reliability may be attained with small increases in cost, while that for the investment budget showed that the optimal value of the objective function increases considerably as the budget decreases. This effect is logical, because of the limitation the investment budget places on new facility location and link construction. We proposed an efficient heuristic based on LP relaxation to solve the proposed mathematical model. Numerical tests showed that the proposed heuristic consistently outperforms CPLEX in terms of solution speed, while still maintaining excellent solution quality.

Our findings raise some questions for future research. First, our heuristic may still require an unacceptably long run time for considerably larger problem instances, so it would be desirable to develop an alternate heuristic capable of solving larger instances in reasonable time. One possible avenue is the development of metaheuristics such as tabu search (TS) and particle swarm optimization (PSO). Second, we considered only a single objective function in this paper; however, considering the RBFLNDP as a multi-objective problem, such as minimizing the operating costs while also maximizing the reliability of system, may find practical application in industries and services.

Table 3: Comparison of performance of heuristic vs. CPLEX

Instance Abbreviation	*N*	*P*	CPLEX				Heuristic				DIFF (%)	
			Value	LB	Gap (%)	Time	Value	LB	Gap (%)	Time	Time (%)	Cost (%)
TP1	5	2	1,475.000	1,475.000	0.000	0.174	1,475.000	1,475.000	0.000	0.173	99.425	100.000
TP2		2	1,964.000	1,817.277	7.471	1.028	1,937.000	1,937.000	0.000	1.027	99.903	98.625
TP3		3	760.000	760.000	0.000	0.195	760.000	760.000	0.000	0.161	82.564	100.000
AVG						0.466				0.454	93.964	99.542
TP4	10	2	26,470.000	26,470.000	0.000	0.902	26,470.000	26,470.000	0.000	0.897	99.446	100.000
TP5		3	19,210.000	19,010.703	1.037	0.833	20,210.000	20,210.000	0.000	0.716	85.954	105.206
TP6		3	51,250.000	47,095.719	8.106	1.297	51,250.000	46,709.762	0.089	1.104	85.120	100.000
AVG						1.011				0.906	90.173	101.735
TP7	15	2	39,880.000	39,291.527	1.476	5.339	39,880.000	39,783.304	0.002	4.068	76.194	100.000
TP8		3	47,564.000	42,861.049	9.888	11.719	44,319.000	43,250.323	0.024	6.779	57.846	93.178
TP9		4	56,250.000	54,250.073	3.555	23.311	56,250.000	56,250.000	0.000	4.563	19.574	100.000
AVG						13.456				5.137	51.205	98.329
TP10	20	3	46,270.000	45,821.508	0.969	66.393	46,270.000	45,832.170	0.009	53.907	81.194	100.000
TP11		4	57,464.000	56,420.465	1.816	84.803	67,012.000	64,402.628	0.039	58.866	69.415	116.616
TP12		5	81,315.000	77,790.430	4.334	66.092	82,546.000	80,215.000	0.028	25.989	39.322	101.514
AVG						72.429				46.254	63.310	106.043
TP13	25	3	64,320.000	61,510.000	4.369	109.099	68,890.000	68,890.000	0.000	69.661	63.851	107.105
TP14		4	113,865.000	103,680.000	8.945	81.197	125,505.000	122,480.894	0.024	52.101	64.166	110.223
TP15		5	141,525.000	141,525.000	0.000	106.862	135,375.000	133,459.066	0.014	49.456	46.280	95.654
AVG						99.053				57.073	58.099	104.327
TP16	30	3	84,650.000	78,353.330	7.438	383.123	83,900.000	82,970.000	0.011	212.089	55.358	99.114
TP17		4	113,203.000	112,181.300	0.903	368.389	123,535.600	123,535.600	0.000	174.173	47.280	109.127
TP18		5	84,858.000	83,605.500	1.476	482.191	92,077.500	92,077.500	0.000	166.196	34.467	108.508
AVG						411.234				184.153	45.701	105.583
TP19	35	3	75,992.000	74,651.593	1.764	583.703	62,167.000	57,693.234	0.072	450.692	77.213	81.807
TP20		4	103,290.000	97,306.902	5.793	634.380	103,290.000	100,587.000	0.026	407.006	64.158	100.000
TP21		5	175,395.000	170,317.200	2.895	532.552	182,751.000	178,687.381	0.022	276.695	51.956	104.194
AVG						583.545				378.131	64.442	95.334
TP22	40	3	93,563.000	89,190.562	4.673	1,138.700	95,480.000	95,480.000	0.000	757.902	66.559	102.049
TP23		4	122,097.000	114,943.500	5.859	1,026.718	135,733.500	135,733.500	0.000	612.342	59.641	111.169
TP24		5	105,918.000	98,899.000	6.627	655.389	105,708.000	105,708.000	0.000	256.917	39.201	99.802
AVG						940.269				542.387	55.133	104.340
TP25	45	4	n/a	n/a	n/a	n/a	85,665.000	85,665.000	0.000	817.790	n/a	n/a
TP26		5	n/a	n/a	n/a	n/a	115,939.500	115,939.500	0.000	1,151.206	n/a	n/a
TP27		6	130,906.000	130,906.000	0.000	1,228.256	135,906.000	130,906.000	0.037	872.010	70.996	103.820
AVG						1,228.256				947.002	70.996	103.820
TP28	50	5	n/a	n/a	n/a	2,500.000	n/a	n/a	n/a	n/a	n/a	n/a
TP29		6	n/a	n/a	n/a	2,500.000	80,266.000	80,266.000	0.000	1,489.721	n/a	n/a
TP30		7	78,761.000	73,384.000	6.827	2,390.350	75,252.000	75,252.000	0.000	1,268.279	53.058	95.545

AVG						2,463.450				1,379.000	53.058	95.545
TP31	55	6	n/a	n/a	n/a	2,500.000	84,550.000	84,550.000	0.000	1,694.246	n/a	n/a
TP32		7	n/a	n/a	n/a	2,500.000	92,670.000	92,670.000	0.000	1,587.678	n/a	n/a
TP33		8	n/a	n/a	n/a	2,500.000	91,740.000	91,740.000	0.000	1,462.357	n/a	n/a
AVG						2,500.000				1,581.427	n/a	n/a
TP34	60	7	n/a	n/a	n/a	2,500.000	125,641.000	115,647.251	0.080	1,942.631	n/a	n/a
TP35		8	n/a	n/a	n/a	2,500.000	121,645.054	115,456.398	0.051	1,802.467	n/a	n/a
TP36		9	n/a	n/a	n/a	2,500.000	119,251.601	112,449.256	0.057	1,745.927	n/a	n/a
AVG						2,500.000				1,830.342	n/a	n/a
Total AVG						831.782				534.790	64.608	101.399

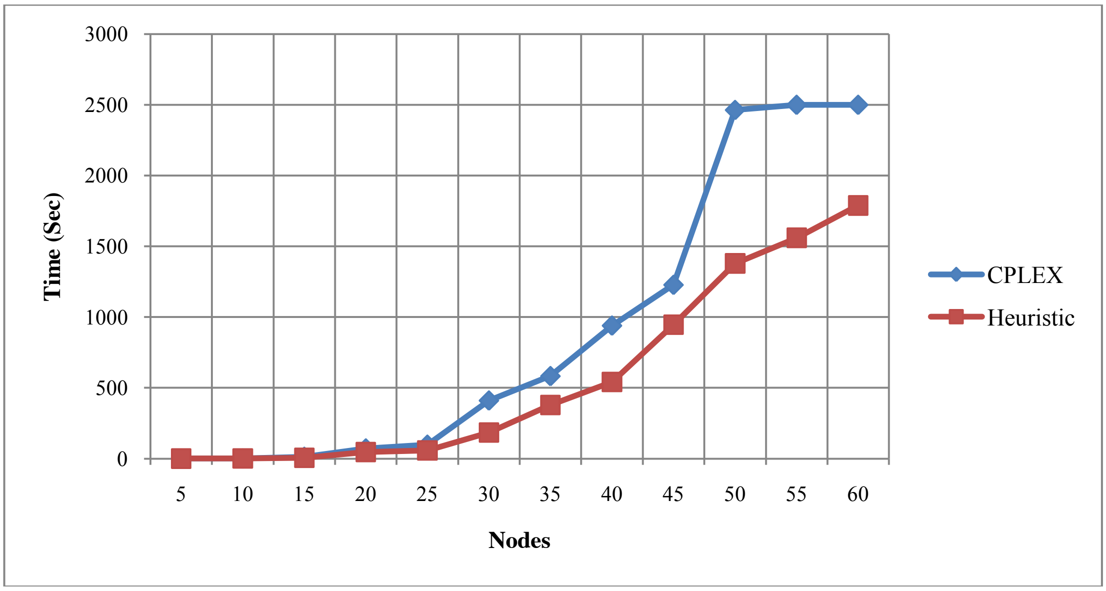

Fig. 6: Run times of proposed heuristic algorithm vs. CPLEX

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